EXAMPLES IN PHYSICS

METHUEN'S TEXT-BOOKS OF SCIENCE

EDITED BY

G. F. GOODCHILD, B.A., B.Sc.,

PRINCIPAL OF THE WANDSWORTH TECHNICAL INSTITUTE

and

G. R. MILLS, M.A.

EXAMPLES IN PHYSICS. By C. E. JACKSON, B.A. *Crown 8vo.* 2*s.* 6*d.*

ELEMENTARY ORGANIC CHEMISTRY. By A. E. DUNSTAN, B.Sc., Head of the Chemical Department, East Ham Technical College. With many Illustrations. *Crown 8vo.*

PRACTICAL CHEMISTRY. By W. FRENCH, M.A., Principal of the Storey Institute, Lancaster. Part I. With 57 Diagrams. *Third Edition. Crown 8vo.* 1*s.* 6*d.* Part II. With 56 Diagrams. By W. FRENCH, M.A., and T. H. BOARDMAN, M.A., Science Master at Christ's Hospital. *Crown 8vo.* 1*s.* 6*d.*

PRACTICAL MECHANICS. By SIDNEY H. WELLS. With 75 Illustrations and Diagrams. *Third Edition. Crown 8vo.* 3*s.* 6*d.*

PRACTICAL PHYSICS. By H. STROUD, D.Sc., M.A., Professor of Physics in the Durham College of Science, Newcastle-on-Tyne. With 115 Diagrams. *Crown 8vo.* 3*s.* 6*d.*

METHUEN'S TEXT-BOOKS OF TECHNOLOGY

EDITED BY

G. F. GOODCHILD, B.A., B.Sc.,

and

G. R. MILLS, M.A.

Fully Illustrated

HOW TO MAKE A DRESS. By J. A. E. WOOD. *Third Edition. Crown 8vo.* 1*s.* 6*d.*

CARPENTRY AND JOINERY. By F. C. WEBBER. *Third Edition. Crown 8vo.* 3*s.* 6*d.*

MILLINERY, THEORETICAL AND PRACTICAL. By CLARE HILL. *Second Edition. Crown 8vo.* 2*s.*

TECHNICAL ARITHMETIC AND GEOMETRY. By C. T. MILLIS, M.I.M.E. *Crown 8vo.* 3*s.* 6*d.*

AN INTRODUCTION TO THE STUDY OF TEXTILE DESIGN. By ALDRED F. BARKER. *Demy 8vo.* 7*s.* 6*d.*

BUILDERS' QUANTITIES. By H. C. GRUBB. *Crown 8vo.* 4*s.* 6*d.*

REPOUSSÉ METAL WORK. By A. C. HORTH. *Crown 8vo.* 2*s.* 6*d.*

EXAMPLES IN PHYSICS

C. E. JACKSON, B.A.

FORMERLY OPEN SCHOLAR OF QUEEN'S COLLEGE, OXFORD; SENIOR PHYSICS MASTER, BRADFORD GRAMMAR SCHOOL

METHUEN & CO.
36 ESSEX STREET W.C.
LONDON

First Published in December 1905

PREFACE

THE object of this work is to supply a collection of suitable problems covering the average Physics course in Secondary Schools. That some such work is needed has of late been a frequent remark among experienced teachers of Physics, and I hope these pages may do something to fill the gap.

The contents—which are new—are divided into two parts; (i) a number of elementary examples intended for younger pupils and adapted to a three years' course under the present regulations of the Board of Education; and (ii) a group of problem papers for the use of more advanced pupils, especially those who are reading for University Scholarships. An attempt has been made to cover the whole ground, and I venture to believe that any pupil who can work through these examples will be well up to the standard of a University Scholarship.

The elementary examples are divided among the various branches of Physics. In Mechanics also a few questions have been inserted, but only such as have experimental work for their basis. The answers have been carefully compiled, and I have to thank my pupil, Mr. B. Lightfoot, who has thoroughly verified them.

The advanced questions have been so constructed as to lead to reasonable results. To this part of the book

no answers have been given, as I am not convinced that these would in themselves be of material advantage. It is my intention, however, to prepare a book of solutions; if any call for such a work should arise.

I should be grateful to hear of any errors which may be detected, or of any improvements which might be introduced in a future edition.

My best thanks are due to my colleagues, Mr. A. Clement Jones, M.A., Ph.D., and Mr. C. H. Blomfield, M.A., B.Sc., who have kindly read the proofs and offered valuable criticism.

C. E. J.

BRADFORD, 1905

CONTENTS

EXAMPLES IN PHYSICS

EXAMPLES IN PHYSICS

I.—MENSURATION

[π may be taken as 3·14, and 1 metre as 39·37 inches.]

1. Express in the metric system (1) a length of 40 yards, (2) a length of 8 feet 4 inches, (3) an area of 15 square inches.

2. How many cubic centimetres are there in a cubic inch?

3. Find the number of yards in a kilometre, and the number of metres in a mile.

4. How many cubic inches are there in a litre?

5. The radius of a circle is 3 feet. Find the length of its circumference (1) to the nearest inch, (2) to the nearest cm.

6. The circumference of a circle is 1 metre. Find the length of its radius (1) in mm., (2) to the nearest tenth of an inch.

7. The distance round a circular field is half a mile. Find the diameter of the field (1) in yards, (2) in metres.

8. The intervals between the minutes on a clock face are 7 inches. Find the diameter of the face.

9. A hoop 3 feet high rolls along the ground. Find how far it travels in 10 complete revolutions, and how many revolutions it makes in 100 yards.

10. The moon describes a circle of radius 240,000 miles about the earth in 28 days. Find the distance it travels in an hour.

11. How many complete circles of radius 6 inches can be formed from a piece of thread 30 yards long? What is the length of the remainder of the thread?

12. Three circles of wire, of radii 6, 10 and 14 cm. respectively, are cut and joined together so as to form one complete circle. Find the radius of this circle.

13. A circular field has a radius of 250 yards. Two men start from a point on the circumference and walk to the point at the other end of the diameter, one going round the field, and the other going directly. If they both walk at the rate of 4 miles an hour, find the difference in the times they take.

14. Find the area of an equilateral triangle of side 12 cm.

15. The circumference of a circle is 87·92 cm. Find its area.

16. A circular field has an area of 1 acre. Find its diameter in yards.

17. The sides of a triangle are 10, 8 and 6 inches long respectively. Find its area (1) in sq. in., (2) in sq. cm.

18. A block of wood is 5 in. long, 6 in. wide and 2 in. thick. Find the area of its surface.

19. Find the area of a parallelogram, the lengths of two adjacent sides being 8 in. and 10 in., and the included angle being 45°.

20. Three semicircles of cardboard, of radii 2 in., 3 in. and 5 in. respectively, are placed together to form a complete curved figure. Find its area and perimeter.

21. From a circular piece of paper of radius 25 cm. a square of side 30 cm. is cut out. Find the area of the remainder.

22. Find to the nearest sq. cm. the area of a rectangular table 1·36 metres long and 93·3 cm. wide.

23. Find the area of a regular hexagon of side 2 in.

24. A regular hexagon is inscribed in a circle of area 78·5 sq. cm. Find its area and perimeter.

25. A quadrilateral has two sides parallel at a distance of 8 in. from each other. If the lengths of these sides are 3 in. and 10 in., find the area of the figure.

26. Find the area of the curved surface of a cylinder 25 cm. high and of radius 6 cm.

27. A cylinder has a height of 15·6 cm. and a radius of 4·4 cm. Find how much paper it will take to cover it.

28. Find the area of the curved surface of a cone 1 ft. high, the radius of whose base is 16 in.

29. The area of the curved surface of a cylinder is 251·2 sq. cm., and the area of each end is 78·5 sq. cm. Find its height and radius.

30. A cylinder 8 cm. high, complete with its two ends, is to be made from a sheet of paper 41·4 cm. long, whose width is equal to the height of the cylinder. Find the radius of the cylinder if there is as little waste as possible.

31. The diagonals of a quadrilateral are 12 cm. and 15 cm. long, and meet at right angles. Find the area of the figure.

32. A circle of radius 10 cm. is cut into 6 equal sectors. Find the area of each.

33. Find the volume of a ball of radius 3 in. What is the area of its surface?

34. The area of the surface of a sphere is 530·66 sq. cm. Find its volume.

35. An orange of radius 2 in. is divided into 10 equal slices. Find the area of the curved portion of each.

36. The circumference of the base of a cylinder is 40·82 cm., and its volume is 1061·32 c.c. Find its height and diameter.

37. Find the volume of a cone whose height is 10 in. and the radius of whose base is 5 in.

38. Find the total area of the surface of a cone 6 cm. high, the radius of whose base is 5 cm.

39. A spherical ball will just fit into a cylindrical vessel of height 8 in. and diameter 3 in. What is the volume of the space not occupied by the ball?

40. A tube has a circular section of diameter 1 mm. and will just contain 1 c.c. of water. Find its length to the nearest mm.

41. A hemisphere of wood of radius 4·5 cm. just fits the base of a cone 12·5 cm. high. Find the volume of the solid.

42. The height of a cone is 24 cm., and its slant height is 30 cm. Find its volume and the total area of its surface.

43. A cylinder is 8 in. high and its radius is 3 in. Find the height of a cone having the same base and volume.

44. Find the volume of a pyramid 25 cm. high, the base of which is an equilateral triangle of side 8 cm.

45. A pyramid 45 cm. high stands on a triangular base, the sides of which are 5, 12 and 13 cm. respectively. Find its volume.

46. Eight drops of water, each 8 mm. in diameter, coalesce to form one drop. Find its radius.

47. The height of a cylinder is 4 cm. and its radius is 3 cm. Find the area of its surface. When it is beaten out into a thin sheet 1 mm. thick, what area will it cover?

48. A cylindrical wire whose volume is 23·55 c.c. has a length of 2083·3 cm. Find its diameter.

49. A piece of gold, having a volume of 2 c.c., is beaten out so as to cover an area of 2 square metres. Find its thickness.

50. A ball of lead, whose volume is 5 c.c., is beaten out into a circular plate of radius 6 cm. Find the thickness of the plate.

---

51. An equilateral triangle is inscribed in a circle of radius 30 cm. Find the area of each of the segments cut off by the sides.

52. Find the area of a quadrilateral $ABCD$, given that $AD = 30$ cm., $AB = 15$ cm., $\angle BAD = 60°$, $\angle DBC = 60°$, $CD = 22{\cdot}5$ cm.

53. From a circular piece of cardboard of radius 25 cm. two circles are cut out, their radii being 10 cm. and 15 cm. respectively. Find the area of the remainder of the cardboard, and its perimeter.

54. A cylinder has hemispheres described on its two ends. Find its height in terms of its radius if the volume of the cylindrical portion is one-half that of the whole body.

55. The height of a truncated cone is 10 in., and its ends are circles of radii 3 in. and 5 in. respectively. Find its volume.

56. A bucket of diameter 1 ft. at the top and 8 in. at the bottom has a height of 15 in. How many times can it be completely filled from a cistern of water 3 ft. long, 2 ft. broad, and $2\frac{1}{2}$ ft. deep?

57. If the radius of the earth at the equator is 3,960 miles, and the earth revolves about its axis once in 24 hours, find the rate at which a point on the equator moves, in miles per hour.

58. Find the volume of a washer of internal and external diameters 2 cm. and 3 cm., and of thickness 1 mm.

59. Find the volume of the space inside a shell formed by concentric spheres of radii 12 cm. and 12·5 cm.

60. A tent has the form of a hexagonal pyramid of height 8 ft., each side of the base being 4 ft. Find the volume of the space enclosed, and the area of the canvas.

61. The area of the curved surface of a cone is twice the area of the base. Find the vertical angle.

62. A sphere of radius 10 cm. is cut into 8 equal parts by planes through a diameter. Find the area of the total surface of each.

63. A sphere is divided into 12 equal parts by planes through a diameter. Prove that the total surface of the parts is four times that of the sphere.

64. A hollow cylinder, whose height is equal to its diameter, has a ball inside it which just fits it. If water is now poured into the cylinder from a full conical vessel of the same height and base, prove that it will be just full.

65. $ABCD$ is a field, the lengths of AB, BC and AD being 100, 150 and 250 yards respectively. BC is parallel to AD, and $\angle BAD = 60°$. Three persons start from A and walk to C; one goes directly, and the others walk round the field in opposite directions. If they all reach C at the same moment, compare their rates of walking.

66. The height of a truncated cone is 30 cm., and its ends are circles of areas 78·5 and 153·86 sq. cm. respectively. Find the area of its curved surface.

67. Water flows into a cistern of capacity 48 cubic feet through a circular pipe of diameter 2 inches. If the rate at which it flows is 2 feet per second, find how long it will take to fill the cistern.

68. A pipe 10 ft. long is made from a sheet of lead $\frac{1}{4}$ in. thick; if its internal diameter is 2 in., find the volume of the metal.

II.—HYDROSTATICS AND MECHANICS

[Unless otherwise stated, densities are expressed in gr. per c.c.]

1. A cube of wood, each side of which is 2·5 cm., weighs 13·75 gr. Find its density.

2. A regular block of wood 10 cm. long, 8·7 cm. wide, and 5 cm. thick, weighs 287·1 gr. Find its density.

3. A plate of zinc is 1 mm. thick and has an area of 48·5 sq. cm. If the weight of the plate is 33·95 gr., find its density.

4. A brass cylinder is 7 cm. long and its diameter is 1·6 cm. The density of brass is 8·3. Find the weight of the cylinder.

5. A sphere of glass weighs 90·43 gr. If its diameter is 4 cm., find its density.

6. Find the height of a cylinder of cork of diameter 3 cm., weighing 14·13 gr., the density of cork being ·25.

7. The density of copper is 8·9. Find the length of a piece of copper wire, ·8 mm. in diameter, weighing 11·36 gr.

8. A cube of ice whose side is 4 cm. is allowed to melt, and the volume of the water is found to be 58·24 c.c. Find the density of ice.

9. Find the thickness of a plate of metal in the form of an equilateral triangle of side 10 cm., weighing 32·73 gr., the density of the metal being 7·2.

10. A flask weighs 8·8 gr. when empty, 33·6 gr. when filled with water, and 28·64 gr. when filled with alcohol. Find the density of alcohol.

3. The density of air is ·00129 when the pressure is 760 mm. What is its density when the pressure is 538 mm.?

54. Air is contained in a cylinder 1 metre long and 5 cm. radius at a pressure of 10 atmospheres. How many litres would the air occupy at atmospheric pressure?

55. A litre of air weighs 1·293 gr. at a pressure of 760 mm. At what pressure will it weigh 1 gr.?

56. Compare the volumes of equal masses of air at pressures of 735 mm. and 672 mm. respectively.

57. A litre flask weighs 39·352 gr. when empty and 40·967 gr. when full of air at a certain pressure. Find this pressure, given that 1 c.c. of air weighs ·001292 gr. at 760 mm.

58. A cubic foot of water weighs 1,000 oz., and the specific gravity of air is ·00128. Find the weight of the air in a room 20 ft. long, 15 ft. wide and 12 ft. high.

59. The barometer stands at 760 mm., and the area of the surface of the mercury in the cistern is 30 sq. cm. Find the total pressure on this area in grammes weight, given that the density of mercury is 13·6.

60. The height of the water barometer is 32 feet, and 1 cubic foot of water weighs 62·5 lb. Find the pressure of the atmosphere in pounds per square inch.

61. The density of sea water is 1·12. Find the pressure in grammes weight per sq. cm. at a depth of 30 metres, neglecting the atmospheric pressure.

62. The height of the water barometer is 34 feet. Find the pressure, in atmospheres, at a depth of 1 mile below the surface of water.

63. The density of alcohol is ·9. Find the pressure in mm. of mercury at a depth of 20 cm. below the surface of alcohol, the density of mercury being 13·5, and the barometric height being 75 cm.

64. Find the pressure in pounds weight on the base of a cubical cistern of side 3 feet, filled with water to a depth of $2\frac{1}{2}$ feet, neglecting atmospheric pressure.

65. At what depth in water is the pressure double of what it is at a depth of 8 inches, the water barometer standing at 33 feet?

66. Water is supplied to a house from a reservoir 810 feet above sea-level. Find in pounds weight per square inch the pressure in a pipe at the top of the house, at a height of 60 feet above sea-level.

67. A tumbler is 5 in. high, and the radius of its base is $1\frac{1}{2}$ in. Find the total pressure on the base when the tumbler is full of water, neglecting the atmospheric pressure.

68. A block of wood floats partly in water and partly in oil of density ·8. If the volumes of the portions immersed in the liquids are as 3 : 2, find the density of the wood.

69. A rod of length 40 cm. is formed of equal lengths of two given kinds of wood, of densities ·66 and ·84. If the rod floats in water, find what length will be above the surface.

70. A piece of lead, weighing 40·68 gr. in air and 37·08 gr. in water, is attached to a piece of wax weighing 4·35 gr., and the two are found to weigh 36·43 gr. in water. Find the densities of the lead and wax.

71. A 50 c.c. flask weighs 9·56 gr. when empty and 46·06 gr. when filled with benzoline. 10·78 gr. of sugar are put into the flask, which is then filled up with benzoline, and the weight is found to be 51·73 gr. Find the density of sugar.

72. A 56 lb. weight weighs only 48·73 lb. in water. Find the specific gravity of the weight.

73. A cylinder of cork, 10 cm. long and of diameter 4 cm., floats in water. If the density of the cork is ·25, find what volume will be immersed.

74. A piece of granite is placed on the upper pan of a Nicholson's hydrometer, and weights are added until the

instrument sinks to the mark. The granite is then removed, and a weight of 8·64 gr. has to be added in order to sink it. The granite is then placed in the lower pan, and in order to sink the hydrometer to the mark, a weight of 5·44 gr. has to be taken out of the pan. Find the density of the granite.

75. The height of the mercury barometer is 760 mm. while a glycerine barometer stands at 813 cm. If the density of mercury is 13·6, find the density of glycerine.

76. If the barometer stands at 76 cm., find the pressure of the atmosphere in pounds per square inch, given that the density of mercury is 13·6, that 1 inch = 2·54 cm., and that 1 lb. = 453·6 gr.

77. A cylindrical vessel 6 feet high and closed at the top is plunged mouth downwards into water. If the height of the water barometer is 34 feet, find to what depth the bottom of the vessel must be plunged in order that the water may rise 2 feet within it.

78. The density of air at a pressure of 76 cm. is ·00129. Find by how much the weight of 10 litres of air alters while the height of the barometer falls from 77 cm. to 74 cm.

79. Find the pressure in kilogrammes weight per square metre at the bottom of a vessel of mercury 1·5 metres deep, the density of mercury being 13·6.

80. The length of a barometer tube is 850 mm. A quantity of air which would occupy a length of 2 cm. of the tube at atmospheric pressure is introduced into the tube, and the level falls to 660 mm. Find the height of the barometer.

81. A body weighs 1856·42 gr. in vacuo and 1595·22 gr. in water. Find its weight in air of density ·00125.

82. A metre rule is balanced about its middle point, and a 5 gr. weight is placed at the 34 cm. mark. Find at what mark a 10 gr. weight must be placed in order that the rule may still balance about its centre.

83. A rod, whose weight may be neglected, has weights of 2 and 6 kg. respectively at its ends. If the length of the rod is 2 feet, find the distance from the smaller weight of the point about which it will balance.

84. Two weights, of 5 and 11 lb. respectively, are attached to the ends of a rod 4 feet long. Find the force required to support the rod, and how far its point of application must be from the larger weight.

85. A rod 5 feet long carries weights of 6 lb. and 14 lb. at its ends. Find the point about which it will balance.

86. A metre rule is balanced about the point marked 47 cm. when a 5 gr. weight is placed at the mark 6·5 cm. Find the weight of the rule.

87. A rod 3 feet long carries weights of 3 lb. and 11 lb. at its ends, and balances about a point distant 1 foot from the larger weight. Find the weight of the rod.

88. A rod 10 feet long, and of negligible weight, is supported at its two ends, and a weight of 150 lb. is placed 3 feet from one end. Find the pressures on the supports.

89. In the preceding example what would be the pressures on the supports if the weight of the rod were 100 lb.?

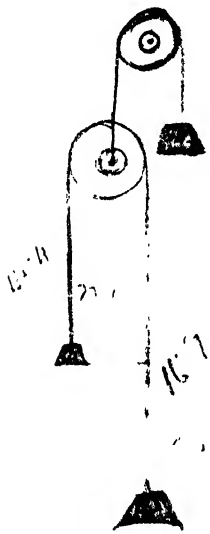
90. Weights of 10 lb. and 25 lb. are placed at the ends of a yard rule. Neglecting the weight of the rule, find what weight must be placed at a point 6 in. from the smaller weight in order that the rule may balance about its middle point.

91. Weights of 2, 4, 6 and 12 lb. respectively are placed 1 foot apart along a yard rule of negligible weight. Find the point about which the rule will balance.

92. The arms of a balance are unequal in length. A body weighs 14·4 gr. when placed in one scale-pan and 16·9 gr. when placed in the other. Find the ratio of the lengths of the two arms, the weights of the scale-pans being neglected.

93. The scale-pans of a balance are unequal in weight while the arms are equal in length. What is the real weight of a body which weighs 40·7 gr. in one scale-pan and 41·3 gr. in the other?

46. A smoked glass plate is held vertically in front of a vibrating fork and is then allowed to fall so that a style attached to one of the prongs of the fork traces out a wavy line on the plate. Starting from a certain point on this line 10 waves can be counted in a distance of 5·4 cm., and 20 waves in a distance of 8·7 cm. Find the frequency of the fork.



PROBLEM PAPERS

Paper I

1. A body of mass 100 kg. is moving at the rate of 30 cm. per second. Calculate its momentum and energy, and find the force required to bring it to rest in 2 seconds.

2. The mass of a body, when the units of length, time and mass are respectively a foot, a second and a pound is 20, and its velocity is 15. Find its momentum and kinetic energy when the units are 4 feet, 2 seconds and 3 pounds respectively.

3. The brass scale of a barometer is correct at 0° C. Find the correction which must be applied to a reading of 756 mm. at a temperature of 14° C. if the coefficients of cubical expansion of mercury and brass are respectively 0·00018 and 0·000054.

4. A calorimeter of mass 120·47 gr., made of a metal whose specific heat is 0·114, contains 220·56 gr. of water at 13° C. Steam is passed into the calorimeter until the temperature rises to 69° C., and the final weight of the calorimeter and the water contained is 364·03 gr. Find the *latent heat of steam.*

5. Find the image of a luminous point placed 25 cm. in front of a concave spherical mirror of radius 15 cm., showing the paths of the rays in a diagram. Find also the magnification of the image.

6. A man looking into a pool sees an object vertically beneath him. He goes back a distance of 10 feet and sees the object in a direction making an angle of 45° with the

horizon. If the height of the man be 6 feet, and the refractive index from air to water be $\frac{4}{3}$, find the depth of the object below the surface.

7. A sphere of radius 10 cm., charged to a potential of 10 electrostatic units, shares its charge with an uncharged sphere of radius 25 cm. Find the changes that take place in the energy of the charged sphere and of the two spheres together.

8. A battery, when connected with a tangent galvanometer of resistance 25 ohms, gives a deflection of 45°; with an extra resistance of 20 ohms in the circuit, the deflection is 30°. What is the resistance of the battery?

9. A coil of radius 10 cm. contains 50 turns of wire. A small magnet placed on the axis of the coil at a distance of 15 cm. from the centre is deflected through an angle of 5° by a current through the coil. Find the magnitude of the current.

10. Given that the velocity of sound in air depends upon the pressure and density only, apply the principle of dimensions to determine the law of variation.

Paper II

1. Find the centre of gravity of a solid formed by a hemisphere standing on the base of a right cone of height 10 inches and vertical angle 60°.

2. A barometer tube contains enough air above the mercury column to occupy a length 2 cm. of the tube at a pressure of 760 mm. If the length of the tube be 950 mm., what length will the air occupy when the pressure is 750 mm.?

3. A kilogramme of copper at 95° C. is introduced into a vessel surrounded by ice and water at 0° C. If the volume of the ice and water be decreased by 8·6 c.c. after the temperature of the metal has fallen to 0° C., find the specific heat of the metal, given that the density of water is unity, that of ice at 0° C. is 0·92, and that the latent heat of fusion of ice is 80 units.

4. A glass weight thermometer weighs 7·5 gr. when empty and 104·6 gr. when filled with mercury at 0° C. At a temperature of 100° C. 1·5 gr. of mercury are expelled; calculate the coefficient of absolute expansion of mercury between 0° C. and 100° C., given that the mean coefficient of cubical expansion of glass between these temperatures is 0·000026.

5. A man stands inside a room, the four walls of which are covered with plane mirrors. Prove that he may see an image of himself by looking in a direction parallel to a diagonal, and show how this image is formed.

6. Rays of light are incident on a parabolic mirror in a direction perpendicular to the axis. Find the form of the caustic curve.

7. Six points are joined in pairs by wires each of resistance 3 ohms. Prove that the equivalent resistance of the framework between any two points is 1 ohm.

8. A magnetic pole of strength 200 units moves with a velocity of 20 cm. per sec. along the axis of a circular coil of wire of mean radius 10 cm., and having 1,000 turns. Find the maximum E.M.F. induced in the coil.

9. An electrometer, when charged by a cell of E.M.F. 1·434 volts, shows a deflection of 77 scale divisions. If the capacity of the quadrants is 28 electrostatic units, find the quantity of electricity corresponding to one division of the scale.

10. A source of sound is emitting a note of frequency 512. Find the pitch of the note heard by an observer (1) when the source is approaching him at the rate of 40 miles an hour, (2) when the observer and the source are moving towards each other at the rate of 60 miles an hour, the velocity of sound in air being 1,100 ft. per sec.

Paper III

1. A hollow sphere of radius 3 feet rotates about a vertical diameter. Find the velocity of rotation in order that a particle inside may rest on the sphere at an angular distance of 60° from the lowest point ($g = 32$ ft. per sec. per sec.).

2. A solid weighs 31·7969 gr. in air, and 33·8707 gr. in water whose density is 0·9996 gr. per c.c., while in a given liquid it weighs 33·5062 gr. If the density of air be taken as 0·001293 gr. per c.c., find the density of the liquid.

3. A leaden bullet at 10° C. strikes a target. If all the energy of the bullet is converted into heat, and the bullet is just melted, find the velocity with which it is moving; given that lead melts at 325° C., has a specific heat 0·03, and latent heat of fusion 5·4, and that Joule's equivalent is $4{\cdot}2 \times 10^7$ ergs.

4. If a gas obeys the law $pv = R(1 + bt)$, where p is the pressure, v the specific volume of the gas, t the temperature on the absolute scale, and R, b are constants, prove that the work done in compressing it is always directly proportional to the heat evolved.

5. A convex lens of focal length 6 cm. is placed 10 cm. in front of a concave spherical mirror of radius 4 cm. Find the position and size of the image of an object 2 cm. high placed 5 cm. in front of the lens.

6. If a ray of light incident on a sphere of glass emerges after two internal reflections in a direction parallel to its original direction, determine the angle of incidence in terms of the refractive index.

7. A Leyden jar has a diameter of 20 cm., and the height of the metal covering is 25 cm. The thickness of the glass is 5 mm., and its specific inductive capacity is 3. Find the work done in charging the jar to a potential of 300 electrostatic units.

8. A battery of E.M.F. 2 volts is in circuit with a resistance of 12 ohms and a wire of resistance 1 ohm; the ends of this wire are also connected to a tangent galvanometer through a resistance of 6 ohms, which includes the resistance of the galvanometer. If the deflection observed is 30°, find the reduction factor of the galvanometer.

9. Two particles, each of mass m gr. and charged with e units of positive electricity, are suspended by light insulating threads of length l cm. from the same point. In the position of equilibrium each thread makes an angle of 45° with the horizon. Prove that $e = l\sqrt{2mg}$.

10. If the density of air at 0° C. and 760 mm. is ·001293 gr. per c.c., the ratio of the specific heats of air 1·4, the specific gravity of mercury 13·6, and the coefficient of expansion of air 0·00366, find the velocity of sound at 15° C. (g = 981 cm. per sec. per sec.).

Paper IV

1. The balance-wheel of a watch keeping correct time at 0° C. is formed of a rim of brass of radius ·8 cm. and of mass 0·02 gr. If the coefficient of linear expansion of brass be 0·000019, find how many seconds the watch will lose per day at 15° C.

2. A particle is dropped from a height of 64 feet above the highest point of a plane inclined at an angle of 30° to the horizon. If the plane be 210 feet long, and the coefficient of restitution $\frac{1}{2}$, prove that the particle will reach the lowest point after three rebounds.

3. A weight thermometer weighs 8·08 gr. when empty, and 18·335 gr. when filled with water at 16° C. On heating to 90° C., 0·30 gr. of water is expelled. If the mean coefficient of cubical expansion of water between 16° C. and 90° C. be 0·000433, find the coefficient of expansion of glass.

4. Find the density of air saturated with water vapour at 20° C. and 755 mm., the pressure of aqueous vapour at 20° C. being that due to 17·4 mm. of mercury, the density of air at 0° C. and 760 mm. being 0·001292 gr. per c.c., and the density of aqueous vapour under the same conditions being 0·000806 gr. per c.c.

5. A luminous point is placed between two plane mirrors inclined at an angle of 60°. Prove that five images will be formed, and draw the rays by which an eye situated between the mirrors will see an image of the object formed by three *reflections at the mirrors.*

6. A person can see distinctly at a distance of 8 inches. What must be the *focal length of an eyeglass which shall* enable him to read a book at a distance of *15 inches?*

7. A tangent galvanometer has a coil of radius 20 cm. with 50 turns of wire. A current passes through it and a copper voltameter, and the deflection observed is 30°. If $H = 0{\cdot}18$ dyne, and an ampere deposits 0·00033 gr. of copper per second, find what mass of copper will be deposited by the current in 20 minutes.

8. A wire is bent into the form of a rectangle, the lengths of whose adjacent sides are 1 foot and 9 inches respectively. The opposite corners are joined by similar wire, and a current enters at one corner and leaves by the opposite corner. If one foot of the wire has a resistance of ·25 ohm, find the equivalent resistance of the framework.

9. A current flows through a circuit containing a tangent galvanometer and a coil of wire immersed in a calorimeter. The coil of the galvanometer has 10 turns of wire of mean radius 13 cm., and the mean deflection is observed to be 69°. The calorimeter initially contains 125 gr. of water at 21° C., and its water equivalent is 17 gr. If the current passes for 7 minutes, and the temperature rises to 38·4° C., find the resistance of the wire.

$H = {\cdot}18$ dyne; $J = 4{\cdot}2 \times 10^7$ ergs; $\tan 69° = 2{\cdot}605$.

10. If the frequency of the note C be 256, find the frequency of the note F♯ on the diatonic scale, and on the scale of equal temperament.

Paper V

1. A rod AB, 10 inches long, rests with its end A in contact with a smooth vertical wall, while the other end is supported by a string attached to a point in the wall 5 inches above A. Prove that the string makes with the wall an angle whose tangent is $\frac{\sqrt{3}}{2}$.

2. A specific gravity bottle of capacity 50 c.c. weighs 20·32 gr. when empty, and 64·25 gr. when filled with benzine. A quantity of sugar, weighing 1·39 gr., is introduced into the bottle, and the bottle is filled up with benzine, its weight being then 64·75 gr. Find the density of the sugar.

3. A constant pressure air thermometer consists of a tube of uniform bore provided with a mercury index and attached to a graduated scale of which the zero is at the bottom of the tube. The thermometer contains a little water, and reads 10·5 at the temperature 15° C. What will be the reading at 40° C. if the pressure of the atmosphere is 760 mm., and the pressure of water vapour is 12·7 mm. at 15° C. and 55 mm. at 40° C., the water being sufficient to saturate the air at both temperatures?

4. A gun weighing 60 tons projects a shot of 500 lb. with a velocity of 2,000 feet per second. Assuming that all the energy of the recoil could be converted into heat, find how much oil it could raise from 10° C. to 30° C., the specific heat of the oil being ·68, and the mechanical equivalent of heat being 1,400 foot-pounds per pound-degree Centigrade.

5. *The image of a point* by reflection in a plane mirror lies always on a fixed straight line. Prove that the line of intersection of the mirror with the plane containing the fixed line and the point always touches a certain conic.

6. A prism of refracting angle 59° is placed on the table of a spectrometer and the slit is viewed in the two positions for minimum deviation. If the telescope readings in these positions be 143° 40′ and 216° 30′, find the refractive index of the prism.

Sin 29° 30′ = 0·49242; sin 47° 42′ = 0·73963;
Sin 47° 43′ = 0·73983.

7. A battery of E.M.F. 3 volts and internal resistance 5 ohms is connected with a tangent galvanometer of resistance 20 ohms, whose coil has 10 turns of wire of mean radius 12 cm. Find the deflection, assuming that the earth's horizontal force is 0·18 dyne.

8. A conductor, consisting of a circular disc of metal of radius 10 cm., is charged with 200 units of electricity. Prove that the force at a point on the axis of the disc distant 5 cm. from its centre is approximately 2·2 dynes.

9. Find the capacity of the earth in microfarads, given that its mean radius is 3,960 miles and that 1 metre = 39·37 inches.

10. A wire of radius ·5 mm., when stretched by a weight of 50 kg., gives a note of frequency 300. Find the weight which must be attached to a wire of the same material, but of double the radius and $\frac{3}{4}$ the length of this one, in order that it may give the octave above.

Paper VI

1. Find the total pressure on the surface of a sphere of radius 5 cm. immersed in a liquid of density 1·5 gr. per c.c., the centre of the sphere being at a depth of 20 cm. below the surface.

2. A mass of 1004·3 grains is required to sink a Nicholson's hydrometer to its mark in water. When a solid is placed in the upper pan a mass of 984·8 grains is required, while when the solid is transferred to the lower pan an additional mass of 7·2 grains has to be added. Find the density of the solid.

3. A calorimeter weighing 77·9 gr. and of specific heat 0·114 contains 167·3 gr. of water at 26° C. 11·5 gr. of ice at 0° C. are added, and the temperature falls to 19·5° C. Calculate the latent heat of fusion of ice.

4. A thermometer tube of uniform bore weighs 10 gr. when empty, 120·5 gr. when the bulb is full of mercury up to the zero mark, and 123·9 gr. when mercury fills it as far as the mark 10 on the stem. Taking the density of mercury as 13·6 gr. per c.c., find the coefficient of expansion relative to glass of a liquid which at 15° C. stands at the mark 11·5 and at 45° C. at the mark 11·9 of the stem.

5. An object placed at a point P in front of a concave spherical mirror gives an image twice the size; when placed at Q the image is magnified three times. If PQ is 1·5 cm., find the radius of the mirror.

6. The refractive index of a prism of angle 60°, found by the method of minimum deviation, was known to be 1·52,

but owing to inaccuracy in reading the scale of the spectrometer the result obtained was 1·53. Show that the error in reading must have been just over 50′.

7. Two points A and B are at potentials 15 and 3 volts respectively, and are connected by a framework of wires of equivalent resistance 5 ohms. The points are now connected by an extra wire of resistance 1 ohm. Prove that the current in it is 2 amperes.

8. A cylindrical magnet of length 6 cm., radius 1·1 cm. and mass 35·4 gr. is vibrating in the earth's field, and the time of a complete oscillation is 9·66 seconds. When placed on one of the arms of a magnetometer with its centre at a distance of 20 cm. from the needle, the deflection observed is 20°, the arms of the magnetometer being at right angles to the magnetic meridian. Find the moment of the magnet and the value of the earth's horizontal force (tan 20° = 0·364).

9. A Leyden jar is charged to a potential of 1,000 volts. When connected with a cylindrical condenser 50 cm. long, of internal and external radii 5 cm. and 5·5 cm. respectively, the potential falls to 900 volts, the dielectric of the condenser being air. Calculate the capacity of the jar.

10. Find Young's modulus for iron, given that the density of the metal is 7·8 gr. per c.c. and that the velocity of sound in it is 5×10^5 cm. per sec.

Paper VII

1. A pendulum consists of a sphere of brass of radius 2 cm. attached to a silk thread of length 20 cm. Prove that the error introduced in the time of oscillation by neglecting the dimensions of the bob is only about 1 in 600.

2. A body slides down a rough plane, for which the coefficient of friction is 0·45, in 4 seconds. If the length of the plane is 200 cm., find its inclination to the horizon.

3. Given that the specific heats of ice and steam are respectively 0·50 and 0·48, and that the latent heats of fusion and vaporisation of water are 80 and 536, find how much ice at $-20°$ C. will be converted into water at 0° C. by the cooling of 3 gr. of steam at 120° C.

4. A plate of metal 20 cm. thick has its opposite faces kept at temperatures 0° C. and 100° C. respectively. If the area of each face is 1·5 square metres, and the thermal conductivity of the metal is ·12 c.g.s. unit, find how much heat passes across the plate per minute.

5. A source of light is placed on an optical bench, and the reading of a pointer attached to it is 60 cm. A lens is placed at the point whose reading is 150 cm., and the image is found to be at the point 183·1 cm. If the radii of the surfaces of the lens are respectively 19 cm. and 34·2 cm., find the refractive index of the material.

6. A prism is cut so that the cotangent of half the refracting angle is equal to the refractive index. Prove that the minimum deviation is the supplement of twice the angle of the prism.

7. The movable disc of an absolute electrometer has a radius of 3 cm., and is at a distance of 4 mm. from the fixed

plate. If the difference of potential between the plates is 1,000 volts, find the force of attraction between them.

8. A copper voltameter and a tangent galvanometer are in series, and a current flowing for 10 minutes deposits 0·06 gr. of copper, and gives a deflection of 25°. If the electrochemical equivalent of copper is 0·00033 gr. per coulomb, find the reduction factor of the galvanometer (tan 25° = 0·466).

9. The sides BC, CA, AB of a triangle are formed of wires of resistances 1, 2 and 3 ohms respectively, and another wire AD of resistance 4 ohms makes sliding contact with BC. A current enters the framework at A and leaves it at D. Prove that when D divides BC in the ratio 1 : 3, just over one-quarter of the current flows along AD.

10. A tuning-fork has a style attached to one of its prongs, which presses against a vertical smoked glass plate. The fork, of frequency 256, is set vibrating, and the plate is allowed to fall, and it is found that 50 waves can be counted in the first 18·7 cm. Find from this experiment the value of g.

Paper VIII

1. A vessel contains water to the depth of 12 inches and mercury to the depth of 8 inches. Find the pressure at a point 15 inches below the surface of the water, given that the specific gravity of mercury is 13·6, and that a cubic foot of water weighs 1,000 oz.

2. The length of a barometer tube above the mercury in the cistern is 781 mm., and when a correct barometer reads 750 mm. the reading on this is 700 mm. Find the reading of the faulty barometer when the pressure is 735 mm.

3. Find the weight of a litre of oxygen saturated with water vapour at 30° C. and 750 mm., given that the densities of oxygen and water vapour at 0° C. and 760 mm. are respectively 0·00143 and 0·00081 gr. per c.c., and that the pressure of water vapour at 30° C. is 31·5 mm.

4. Prove that for a gas whose pressure, specific volume and absolute temperature are connected by the relation $pv = R(1 + ct)$, where R and c are constants, the difference of the specific heats at constant pressure and constant volume is equal to $\frac{Rc^2t}{1 + ct}$.

5. The feet of a spherometer form the corners of an equilateral triangle of side 11·1 cm. When the spherometer is placed on a plane surface, the reading of the scale is 24·99 mm., while on a spherical surface it reads 28·27 mm. Find the radius of the spherical surface.

6. A glass prism of refracting angle 60° has a refractive index of 1·5 for light of mean refrangibility. Prove that the

increase in minimum deviation corresponding to a small increase in refractive index bears to the increase in refractive index a ratio approximately equal to 1·51.

7. A tangent galvanometer of resistance 10 ohms shows a deflection of 45° when connected to a battery through a resistance of 341 ohms. When the resistance is altered to 985 ohms, the deflection is only 20°. Find the resistance of the battery, given that tan 20° = 0·364.

8. A coil of 200 turns and mean radius 25 cm. rotates 10 times a second about a vertical diameter. If the intensity of the earth's horizontal force is 0·18 dyne, calculate the maximum E.M.F. induced in the coil.

9. Find the moment of a magnet of length 7 cm., breadth 13 mm. and mass 45 gr., which makes 10 oscillations in 127 seconds in the earth's field, the intensity of the earth's horizontal force being 0·18 dyne.

10. If the velocity of sound in air at normal temperature and pressure be 330 metres per second, the ratio of the specific heats of air 1·4, the density of mercury 13·6 gr. per c.c., and g = 981 cm. per sec. per sec., prove that the height of the homogeneous atmosphere is nearly 5 miles.

1 foot = 30·45 cm.

Paper IX

1. Find the pressure at a point inside an air-bubble of radius 0·3 mm. at the bottom of a vessel of water 20 cm. deep, the atmospheric pressure being that due to 760 mm. of mercury, the density of mercury being 13·6 gr. per c.c., and the surface tension of water 78 c.g.s. units.

2. A corner of a square is cut off along the line bisecting two adjacent sides. Find the centre of gravity of the remainder.

3. Calculate the mean square of the velocities of the molecules of air at 15° C. and 760 mm., given that the density of air at 0° C. and 760 mm. is 0·001294 gr. per c.c., and that its coefficient of expansion is 0·003665.

4. *In finding the latent heat of fusion of ice 20 gr. are melted by 250 gr. of water in a calorimeter of water equivalent 15 gr. If the readings of the thermometer are liable to an error of* ·1° C., *prove that the greatest error which can occur in the value of the latent heat is nearly 3 calories.*

5. Two plane mirrors are inclined at an angle of 30° to one another. Prove that a ray of light incident upon one of them in a plane perpendicular to their line of intersection and in a direction parallel to the plane bisecting the angle between the mirrors will, after three reflections at each mirror, be parallel to its original direction.

6. A ray passes through a prism of refractive index 1·5 in such a way that the incident and emergent rays make angles of 30° and 60° respectively with the two faces. Prove that the direction of the ray within the prism makes with one face an angle equal to the angle of the prism.

7. How many cells of E.M.F. 1·1 volts and resistance 2·5 ohms will be required to send a current of $\frac{1}{4}$ ampere through an external resistance of 50 ohms?

8. Two spheres of radii 6 cm. and 5 cm. respectively are charged with 30 and 20 units respectively of positive electricity. The two are then joined by a wire. Find the changes that take place in the charges and the electrical energies of the spheres.

9. Prove that the tangents of the deflections of a compass needle placed at the centres of coils constructed of a given length of the same wire when the same current traverses each are approximately inversely proportional to the squares of the radii of the coils.

10. A string 2 ft. long, weighing $1\frac{1}{2}$ oz., is stretched by hanging a weight of 200 lb. to it. Find the frequency of the fundamental note emitted when the string vibrates transversely.

Paper X

1. A body is projected vertically upwards with a velocity of 20 feet per second. If the coefficient of restitution between the body and the ground be $\frac{1}{3}$, find what time will elapse before it comes to rest.

2. A body is executing simple harmonic motion in a straight line. If k^2 is its acceleration when at a distance of 4 cm. from its mean position, prove that the periodic time is $\frac{4\pi}{k}$.

3. Two equal rods, of brass and steel respectively, are placed side by side, and carry a vernier scale by means of which their relative expansion can be observed. If a rise of 50° C. in temperature causes a relative expansion of ·05 mm., find the length of each rod, given that the coefficients of linear expansion of brass and steel are respectively 0·000019 and 0·000012.

4. A calorimeter of water equivalent 10 gr. is filled with water at 80° C., and the time taken for the temperature to fall to 75° C. is 4 minutes. When filled with another liquid the time taken for the temperature to fall through the same range is 130 seconds. The weight of the water is 50 gr., and that of the liquid 40 gr. Find the specific heat of the liquid.

5. What must be the focal length of a lens which, when held at a distance of 15 cm. from the eye, magnifies an object five times?

6. In Rumford's photometer the rod is distant 10 cm. from the screen, and two sources have to be placed at

distances of 20 cm. and 48 cm. respectively from the rod in order that the shadows thrown on the screen may be equally dark. Compare the illuminating powers of the two sources.

7. Twenty-four lamps, in four parallel rows of six each, are connected to the leads from a dynamo. The difference of potential between the leads is 230 volts, and each lamp takes ·25 ampere. Find the equivalent resistance between the leads.

8. Find the energy of a battery of 20 Leyden jars, the inner coatings of which are connected and raised to a potential of 3,000 volts, while the outer coatings are earthed; the radius of each jar being 10 cm., the height of the tinfoil covering 40 cm., and the glass of each being 3 mm. thick and of specific inductive capacity 7.

9. In an attracted disc electrometer the area of the movable plate is 50 sq. cm., the distance between the plates is 4 mm., and the difference of potential 1,000 volts. Find, in dynes, the force of attraction between the plates.

10. Assuming that the frequency of the note C is 256, find the number of beats that would be heard per second between the two notes which represent the major sixth above C (1) on the diatonic scale, (2) on the scale of equal temperament.

Paper XI

1. Find the total pressure on a square immersed vertically in water, the upper and lower edges being parallel to the surface and at depths 25 cm. and 40 cm. respectively.

2. A circular table weighing 50 lb. stands on three legs placed at equal intervals round its circumference. If the diameter of the table is 5 feet, find the least weight which, when placed on the table, will overturn it.

3. The coefficient of cubical expansion of mercury is 0·00018, and that of glass is 0·000027. Find what length of a tube of glass 30 cm. long must be filled with mercury in order that the volume of the space above it may be the same at all temperatures.

4. Find the height through which water at 50° C. must fall in order that its temperature may rise to 60° C., assuming that all the energy of the fall is converted into heat, and that 1,390 foot-pounds of work have to be done in order to raise the temperature of 1 lb. of water 1° C.

5. The two positions of a convex lens which give an image of an object on a screen 100 cm. from the object are distant 12 cm. from each other. Find the focal length of the lens, and the magnification produced in each case.

6. Find the dispersion produced by a thin glass prism of angle 10°, the refractive indices of which for red and violet light are respectively 1·627 and 1·643.

7. A current is passed through a water voltameter, a copper voltameter, and a tangent galvanometer in series. After passing for 700 sec. it is found that 23·5 c.c. of

sited, while the mean deflection of the galvanometer has been 34°. If the electro-chemical equivalent of hydrogen is 0·00001038 gr. per coulomb, find the electro-chemical equivalent of copper, and the reduction factor of the galvanometer.

Tan 34° = 0·674; 1 c.c. of hydrogen weighs ·0896 gr.

8. A battery of internal resistance 4 ohms has its terminals connected to a circuit of wire of resistance 16 ohms. Find by how much the closing of the circuit will affect the difference of potential between the terminals.

9. The area of the surface of a Leyden jar is 200 sq. cm., and the dielectric is formed of material 5 mm. thick, of specific inductive capacity 7. The internal coating is charged to a potential of 300 electrostatic units, while the outer coating is earthed. Find the loss in the energy of the jar if the outer coating is insulated and the knob touched.

10. Find the specific heat of air at constant volume, given that the velocity of sound in air at 20° C. and 760 mm. is 344 metres per second, the density of air at the same pressure and temperature 0·001294 gr. per c.c., the coefficient of expansion of air 0·003665, and the mechanical equivalent of heat 42×10^6 ergs.

Paper XII

1. Find the kinetic energy of a thin hoop, weighing 10 kg. and of radius 25 cm., rolling at the rate of 300 cm. *per sec.*

2. A sphere of metal of specific gravity 7·8 is dropped into a vessel containing mercury and water. If the specific gravity of mercury is 13·6, find how much of the sphere will be immersed in it.

3. How many units of heat will pass per minute across a plate of copper 90 cm. long, 65 cm. broad and 8·4 cm. thick, when its opposite faces are at the temperatures 5° C. and 50° C., the conductivity of copper being 0·95 c.g.s. unit?

4. A piece of metal weighs 277·005 gr. in vacuo, 241·605 gr. in water at 4° C., and 243·528 gr. in water at 80° C. If the density of water at 80° C. is 0·972 gr. per c.c., find the coefficient of expansion of the metal.

5. Find the least value of the refractive index of a prism which gives a minimum deviation of 60°.

6. A luminous point P is situated between two plane mirrors inclined at an acute angle. A ray from P is reflected at each mirror and then returns to P. Prove that the line bisecting the angle between the two rays through I passes through the intersection of the mirrors.

7. Three spheres, of radii 1 cm., 1·5 cm. and 2 cm respectively, are charged to potentials 3, 4 and 5 units. If they are then connected by a wire, find the charge and electrical energy of each sphere.

8. A battery is in circuit with a resistance box and a shunted galvanometer. The deflection is noted, and after

removing the shunt it is found that the resistance has to be trebled in order to give the same deflection. Prove that, neglecting the resistance of the battery, the resistance of the shunt is half that of the galvanometer.

9. A skeleton cube is formed of 12 wires of equal resistance. A current of 2 amperes enters at one corner and leaves at the other end of the diagonal through that corner. Find the current in each of the wires.

10. A cylindrical vessel of glass, 2·5 inches in radius, is partly filled with water, and it is found that when the length of the column of air above the water is 10·5 inches the note emitted by a tuning-fork sounded just above it is loudest. Taking the velocity of sound in air as 1,100 feet per second, find the frequency of the fork.

Paper XIII

1. In using a Nicholson's hydrometer it is found that 27·8 gr. are required to sink it in water to the mark. A solid is placed first in the upper pan and then in the lower pan, and the weights required to sink the instrument are respectively 14·6 gr. and 18·7 gr. Find the density of the solid.

2. $ABCD$ is a rhombus, the angle A being 45° and the length of each side being 20 inches. Find the centre of gravity of masses of 2, 3, 4 and 6 lb. placed at A, B, C and D respectively.

3. A straight vertical tube, of cross-section 5 sq. cm., is closed at the lower end, and is filled with air which supports a closely fitting piston of mass 20 gr. The temperature of the air is 25° C. If now an additional mass of 30 gr. be placed on the piston, find how much the temperature must increase in order that the piston may be brought back to its former position, given that the height of the barometer is 750 mm., the density of mercury 13·6 gr. per c.c., the coefficient of expansion of air 0·003665, and g being 981 cm. per sec. per sec.

4. The latent heat of fusion of ice is 80, and its density is 0·93 gr. per c.c. at 0° C. A piece of metal weighing 50 gr. is heated to 75°, and is placed in a Bunsen calorimeter. If the contraction that takes place is 0·469 c.c., find the specific heat of the metal.

5. A convex lens of focal length 20 cm. is placed in front of a concave spherical mirror of radius 15 cm. Find the position of the image of a luminous point placed 30 cm.

in front of the lens, and draw a diagram showing how the image is formed.

6. Prove that the greatest angle which an incident ray can make with one face of a prism of angle 60° and refractive index 1·5 in order that it may emerge at the other face is about 62°.

7. Two small insulated spheres, of radii a cm. and b cm., are charged with p and q units respectively of positive electricity. If they are connected by a wire of negligible capacity, find how much electricity will flow along it.

8. A current of ·1 ampere flows through a circular coil of wire of 20 turns and of mean radius 15 cm. Find what force will be exerted by the current on a magnetic pole of strength 5 units placed at the centre of the coil.

9. A steady current is passed for one hour through a copper voltameter and also through a galvanometer of resistance 8 ohms, shunted by a resistance of 0·5 ohm. The amount of copper deposited is 0·165 gr., and the mean deflection of the galvanometer is 50°. Find the strength of the current and the reduction factor of the galvanometer, given that one ampere deposits 0·00033 gr. of copper per second, and that tan 50° = 1·192.

10. Assuming that the period of vibration of a stretched string depends on its length, tension and linear density, apply the principle of dimensions to find the law of variation.

Paper XIV

1. A cylinder 4 cm. long and 1 cm. radius weighs 40 gr. in air and 30 gr. in a given liquid. Find the density of the cylinder and of the liquid.

2. Given that the compressibility of water per atmosphere is 5×10^{-5} in c.g.s. measure, calculate the density at the bottom of a reservoir 25 metres deep, given that the height of the water barometer is 75 cm.

3. If the zero on the absolute scale of temperature is $-273°$ C., find the volume at 15° C. and 748 mm. of a mass of air which occupies a volume of 157·9 c.c. at 95° C. and 770 mm.

4. A weight thermometer of glass contains 50 gr. of mercury at 0° C., and when placed in a bath it is found that 1·05 gr. are expelled. If the coefficients of cubical expansion of mercury and glass are respectively 0·00018 and 0·000026, find the temperature of the bath.

5. The image of a gas-flame at a height of 4 ft. 3 in. above a table is thrown on a table by a convex lens of focal length 10 in. Find the position of the lens.

6. A hollow sphere of radius 8 cm. is polished at the ends of a diameter PQ. A luminous point is placed on the line PQ at a distance of 2 cm. from the centre of the sphere. Prove that the distance between the images of the point formed by reflection at P and Q is $2\frac{2}{3}$ cm.

7. A circuit contains a galvanometer and a battery, the total resistance being 6·5 ohms. The deflection of the galvanometer is 40°. A piece of iron wire is introduced into the circuit and the deflection falls to 25°. If the diameter

of the wire is 0·8 mm. and the specific resistance of iron is 9,750 c.g.s. units, find its length.

Tan 40° = 0·839; tan 25° = 0·466.

8. Two points, joined directly by a wire of resistance 1 ohm, are also connected to the terminals of a 2-volt battery of internal resistance 5 ohms through a resistance of 500 ohms, and to a galvanometer of resistance 10 ohms through a resistance of 50 ohms. Find the current in the galvanometer.

9. The movable plate of an absolute electrometer is at a potential of 100 electrostatic units and at a distance of 20 cm. from the other plate, which is earthed. The area of the plate is 100 sq. cm. A plate of glass 2 cm. thick, and of specific inductive capacity 7, is introduced between the plates: find the force acting on the movable plate.

10. An organ pipe, sounded with a tuning-fork of frequency 512, gives 10 beats per second, the frequency of the fork being less than that of the pipe. Find by how much the length of the pipe must be altered in order that the two may give the same note, given that the velocity of sound in air is 332 metres per second.

Paper XV

1. A cylinder of lead, whose length is 15 cm. and radius 2·5 cm., is suspended in water by a string. If the density of the lead is 11·5 gr. per c.c., find the tension in the string.

2. Find the velocity with which a particle must be projected at an angle of 60° with the horizon, so as just to clear a wall 50 feet high at a distance of 75 feet from the point of projection. 74 ft/sec nearly. 74·9 ft/sec

3. Assuming that a cubic foot of water at its point of maximum density weighs 1,000 oz., that the density of water at 100° C. relative to its greatest density is 0·954, and that 1 cubic foot of water at 100° C. occupies 1,680 cubic feet when converted into steam at the same temperature and at 760 mm. pressure, find the weight of a cubic foot of this steam, and also what fraction of the latent heat of steam is spent in doing external work.

4. 1 gr. of hydrogen at 0° C. and 760 mm. occupies a volume of 11·16 litres. Find the temperature at which its volume will be 16 litres, when subjected to a pressure of 1,200 grammes weight per sq. cm., the density of mercury being 13·596 gr. per c.c., and g being 981 cm. per sec. per sec.

5. A ray of light is incident on a refracting sphere. The refracted ray is partly reflected at the inner surface of the sphere and partly refracted out into the air. Prove that these two parts will be at right angles if the tangent of the angle of incidence is equal to the refractive index.

6. A microscope is focussed on a speck at the bottom of an empty beaker. Water is then poured into the vessel

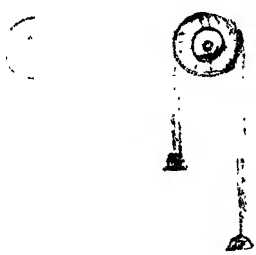
until it stands at a height of 5 cm. If the refractive index of water is $\frac{4}{3}$, find the distance through which the microscope must be moved in order that the speck may again appear in focus. Does this distance depend on the power of the microscope?

7. Two points at given difference of potential are joined by n wires of equal resistance. Prove that the heat generated when the wires are arranged in parallel is equal to n^2 times the heat generated when they are arranged in series.

8. A circuit contains a battery, a resistance of 20 ohms, a 1-ohm coil and an unknown resistance. A galvanometer of resistance 5 ohms is connected to the terminals of the coil through a resistance of 860 ohms, and the deflection is observed. The galvanometer is then connected to the ends of the unknown resistance, and it is found that in order to produce the same deflection as before only 30 ohms are required in the circuit. Find the value of the unknown resistance.

9. Three thin magnets, AB, BC, CD, each of length 10 cm. and moment 250, are placed so as to form three adjacent sides of a regular hexagon, the north poles of the magnets being A, B, C respectively. Find the magnitude and direction of the force exerted by the combination at the centre of the hexagon.

10. A wire of length 54·5 cm. is stretched by a weight of 43·6 kg. If 409·6 metres of the wire weigh 1 kg. and the frequency of the note C is 256, prove that when the wire is bowed it will emit the note G.



Paper XVI

1. Three liquids of densities 0·8, 1·2 and 13·6 gr. per c.c. respectively are contained in a vessel, the depth of the middle one being 12 cm. A rod of density 8·8 gr. per c.c. and length 40 cm. floats in the liquids so as to be wholly immersed. Find the length of the rod in the densest liquid.

2. Find to what depth a diving-bell must be lowered into water in order that the volume of the air contained may be diminished by one-quarter, the length of the bell being 3 metres, the atmospheric pressure 760 mm. and the specific gravity of mercury 13·6.

3. If the barometric height be 760 mm., find the pressure of the atmosphere in pounds per square inch. If the temperature increases by 50° C. while the height of the barometer remains the same, find the alteration in the pressure given that 1 c.c. of mercury at the first temperature weighs 13·6 gr.; coefficient of expansion of mercury = 0·00018; 1 lb. = 453·59 gr.; 1 metre = 39·37 inches.

4. Compare the volumes of equal masses of air (1) at − 20° C. and 770 mm., (2) at 40° C. and 787 mm.

5. A bright point is situated on one wall of a room 12 ft. long, and a convex lens of 10 in. focal length and 3 in. diameter is placed 2 ft. from the wall so that its centre lies on the normal to the wall through the luminous point. Find the area of the circle of light thrown on the opposite wall.

6. Find the refractive index and the angle of a prism from the following data: readings of the telescope on the spectrometer when in position for viewing the slit by reflection at the two faces, 50° 20′ and 170° 20′ 30″; readings of

telescope for minimum deviation on each side of the line of direct vision, 166° 3′ 30″ and 251° 51′.

Sin 60° 0′ 15″ = 0·86606; sin 51° 27′ = 0·78206.

7. Two concentric spheres A and B of radii 1 cm. and 2 cm. respectively are placed at a great distance from two other concentric spheres C and D of radii 3 cm. and 4 cm. The outer spheres are connected by a wire, and a charge of 18 units is given to A, while C is earthed. Prove that the sphere D is at unit potential.

8. How many cells of E.M.F. 1·08 volts and internal resistance 1·5 ohms will be required to send a current of $\frac{1}{3}$ ampere through an external resistance of 15 ohms?

9. A suspended magnet makes 10 oscillations in 117·6 seconds at a place where H, the earth's horizontal intensity, is 0·189 dyne. How many complete oscillations will it make per minute at another place where $H = 0{\cdot}214$?

10. Find the velocity of sound in hydrogen, given that its velocity in air is 332 metres per second, and that the volumes occupied by a gramme of hydrogen and of air are respectively 11·161 and 0·774 litres.

Paper XVII

1. A body is let fall from a height of 32 feet above a horizontal plane; find how long it will be before it comes to rest, the coefficient of restitution between the body and the plane being $\frac{3}{4}$.

2. A hoop of radius 2 ft. rolls along the ground with a velocity of 10 ft. per sec.; if its mass be 5 lb. and its thickness be neglected, find its kinetic energy.

3. A glass globe of capacity 500 c.c. is full of air at 0° C. If the air is allowed to expand at constant pressure, find what volume will be expelled at 100° C., reckoned at the pressure of 760 mm. and at 0° C., given that the coefficient of cubical expansion of glass is 0·000027, and that of air 0·003665.

4. An iron boiler of thickness 3 cm. has a heating surface of 1 square metre. Assuming that the temperature of the furnace is 300° C. and that the boiler contains water at 100° C., find how many kilogrammes of water are converted into steam per minute, the thermal conductivity of iron being 0·15 c.g.s. unit.

5. If x is the minimum deviation for a prism of angle A, and y that for a prism of angle $2A$, prove that $2 \sin \frac{y-x}{4} \cos\left(A + \frac{y+x}{4}\right) = \sin \frac{x}{2}$.

6. A speck in a sphere of glass 10 inches in diameter is viewed so that it is in a line with the eye and the centre of the sphere. If the speck appears to be 2 inches from the nearer surface of the sphere, and the refractive index of the glass is 1·5, find its real position.

7. A current, which produces a deflection of 70° in a tangent galvanometer whose reduction factor is 0·038, is sent through a resistance of 15 ohms placed in a calorimeter containing water. If the mass of water is 200 gr. and the water equivalent of the calorimeter is 15 gr., find the rise of temperature after 10 minutes.

$J = 4{\cdot}2 \times 10^7$ ergs; $\tan 70° = 2{\cdot}75$.

8. A cell of internal resistance ·5 ohm works through an external resistance of 2 ohms. Find the resistance of a second cell of equal E.M.F. which, when joined up in series with the first and working through the same resistance, produces the same current as before.

9. A magnet of length 8 cm. is placed on the arm of a magnetometer with its centre at a distance of 20 cm. from the needle, the arms of the instrument being at right angles to the magnetic meridian. Calculate the percentage of error introduced into the value of the moment of the magnet determined on the supposition that the force exerted by the magnet varies inversely as the cube of the distance of its centre from the needle.

10. If the frequency of the note C be taken as 256, find the number of beats heard per second when it is sounded simultaneously with the note B below it.

Paper XVIII

1. A piece of copper weighing 30 gr., and a piece of iron weighing 25 gr., are suspended together in water from the arm of a balance. Find the weight which must be placed in the other scale-pan to secure a balance, given that the densities of the metals are respectively 8·8 and 7·8 gr. per c.c.

2. A piece of paper in the shape of a rectangle of length 8 inches and breadth 5 inches is folded along the line joining the middle points of two adjacent sides. Find the centre of gravity of the paper when so folded.

3. A quantity of air saturated with water vapour at 100° C. is contained in a closed vessel. When heated to 150° C. the pressure is found to be equal to that of 1·35 atmospheres. Find the pressure of the given volume of dry air at 15° C. and at a pressure of 1 atmosphere.

4. A spherical ball of iron of diameter 4·68 cm. at 15° C. rests on a ring of zinc, the internal diameter of which is 4·67 cm. at the same temperature. Find the temperature to which both must be raised in order that the ball may just pass through the ring, given that the coefficients of linear expansion of iron and zinc are respectively 0·000012 and 0·000030.

5. How would you arrange two thin convex lenses, of focal lengths 19·5 cm. and 19·22 cm. respectively, so as to be equivalent to a single thin lens of focal length 10 cm.?

6. A prism of refractive index 1·5 is cut so that the minimum deviation is equal to the refracting angle. Prove that each of these is about 83°.

7. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is without twist. The upper end of the wire is turned through an angle of 150°, and the magnet is deflected through 30°. Find through what angle the upper end must be turned in order that the magnet may make an angle of 60° with the meridian.

8. Calculate the specific resistance of a wire from the following data: weight of wire = 0·558 gr.; weight of wire in water = 0·490 gr.; length of wire = 43·2 cm.; resistance of wire = 1·077 ohms.

9. A coil, having 50 turns of mean radius 20 cm., carries a current which deflects a compass needle at its centre through 20°, the coil being at right angles to the magnetic meridian. If tan 20° = 0·364, find the strength of the current (H = 0·18 dyne).

10. A wire of length 100 cm., weighing 0·8 gr., is under tension, and when plucked gives a fundamental note of frequency 240. Find the amount of the stretching force.

Paper XIX

1. A body at the equator weighs 10 lb. What would be its weight if the earth rotated only half as fast as it does?

[Radius of earth = 3,960 miles; g = 32 ft. per sec. per sec.]

2. Two masses, of 5 and 10 lb. respectively, connected by a string 3 feet in length, rest on the edge of a tower. The smaller mass is allowed to fall from its position; find the velocity with which the two will be moving 10 seconds later.

3. A quantity of gas was found to occupy a volume of 98·52 c.c. at a temperature of 35° C. and at 756 mm. pressure. At 10° C. and 740 mm. the volume occupied was 92·22 c.c. Find the coefficient of expansion of the gas.

4. A mass 80 gr. of copper at 98° C. is placed in a copper calorimeter weighing 30 gr., which contains 50 gr. of a certain liquid at 15° C. The resulting temperature is 32·7° C. If the specific heat of copper is 0·095, calculate the specific heat of the liquid.

5. A convex and a concave lens, each 12 inches in focal length, are placed coaxially at a distance of 5 inches apart. Find the position and size of the image of an object 3 inches high placed 20 inches in front of the convex lens.

6. It is required to make an achromatic combination of two lenses whose mean refractive indices are 1·52 and 1·62 respectively, and whose dispersive powers are in the ratio 2 : 3. If the focal length of the combination is 10 cm., find the focal length of each lens.

7. A battery of 70 cells, each of E.M.F. 1·4 volts and resistance 0·5 ohm, is used to send a current through a wire

of resistance 20 ohms. Find the value of this current if the cells are arranged in 7 parallel groups of 10 each; and find what arrangement will give the greatest possible current in the wire.

8. A compass needle makes 10 oscillations in 2 min. 44 sec. under the action of the earth's force alone. Under the combined action of the earth and a magnet it makes 10 oscillations in 2 min. 18 sec.; while when a second magnet is substituted for the former the time is 2 min. 2 sec. Compare the moments of the two magnets.

9. A galvanometer has a coil containing 50 turns of wire of mean radius 20 cm. If the intensity of the earth's horizontal force is 0·18 dyne, find the E.M.F. of a battery which will deflect a magnet placed at the centre of the coil through 45°, the resistance of the circuit being 7·5 ohms.

10. A wire 1 metre long gives the note C of frequency 256 when vibrating transversely. Find the frequency of the note emitted by a wire of the same material 25 cm. long and half as thick as the former.

Paper XX

1. A bullet weighing 10 gr. is shot into a piece of wood weighing 5 kg., which hangs by a string 1 metre long. If the wood swings until the string makes an angle of 60° with the vertical, find the velocity of the bullet.

A cylinder of length 20 cm. is formed partly of a metal whose density is 21 gr. per c.c., and partly of another metal whose density is 7·8 gr. per c.c. If the body floats in mercury of density 13·6 gr. per c.c. so that it is just immersed, find the lengths of the two parts.

3. Prove that the coefficient of cubical expansion of a substance is approximately equal to three times the linear coefficient, and find the error involved in this approximation in calculating the cubical expansion through 250° C. of a metal whose linear coefficient is 0·0000196.

4. A gramme of air at 0° C. and 760 mm. occupies a volume of 0·772 litre; the difference of the specific heats of air at constant pressure and constant volume is 0·0688. Taking the density of mercury as 13·596 gr. per c.c. at 0° C., find the value of the mechanical equivalent of heat, given that the coefficient of expansion of air is 0·003665.

5. Compare the focal lengths of a lens in air and in a certain liquid, given that the refractive index from air to glass is 1·53, and that the index from air into the given liquid is 1·41.

6. Prove that there are two positions for an object placed in front of a concave spherical mirror so that the image may be of given size. If the radius of the mirror is 25 cm. and the required magnification is 3, find the distance between the two positions of the object.

7. Four wires, AB, BC, CD, DA, of resistances 1, 2, 3 and 4 ohms respectively, are joined to form a complete circuit. A current of 1 ampere enters at A and leaves at C; find the currents in the branches AB and AC, and calculate the equivalent resistance of the framework.

8. A coil is held at right angles to the magnetic meridian, and is rotated suddenly through an angle of 60°; it is then held in this position and rotated through an angle of 30° in the same direction. Prove that the quantities of electricity passing through the coil in the two cases are equal.

9. Two points are connected to the terminals of a battery of E.M.F. $2e$ and resistance r_1, and also to another of E.M.F. e and resistance r_2, the resistances in the wires being neglected. The points are also joined by a wire of resistance S. Prove that the current through the first battery is double that through the second if $2(r_1 - r_2) = 3S$.

10. A wire, stretched by a weight of 24 lb., emits the note G when vibrating transversely. What weight must be attached to a wire of the same material but double the length of the former in order that it may give the note E, the frequency of the note C being 256?

Paper XXI

1. A train weighing 200 tons is travelling at the rate of 60 miles an hour. If the resistance to the motion is 10 lb. per ton, what is the horse-power of the engine?

2. A string passing over a smooth pulley carries a mass of 500 gr. at each end. A mass of 20 gr. is attached to one of them, and is taken off again after 3 seconds. Find how far the system will move in the next 2 seconds.

3. 20 gr. of metal at 96° C. are placed in a Bunsen's ice-calorimeter, and after the temperature of the metal has fallen to 0° C. the position of the end of the mercury column indicates a contraction of 0·207 c.c. If the density of ice at 0° C. is 0·916 gr. per c.c., and the latent heat of fusion of ice is 80·03, find the specific heat of the metal.

4. A slab of iron, 30 cm. long, 20 cm. broad and 10 cm. thick, has a quantity of ice on its upper surface, while its lower surface is exposed to a source of heat at a temperature of 100° C. If 810 gr. of ice are melted in 1 minute, find the thermal conductivity of the metal.

5. The focal length of a lens is found by the following method. The object is at a fixed point on the optical bench, and the position of the microscope with which the image is viewed is also fixed, while the lens is moved about so as to determine the two positions in which the image is in focus. If the shift of the lens between these positions is 200 mm., and the two images are respectively 7·44 mm. and 2·46 mm. high, find the focal length of the lens.

6. Three sources of light of respective candle-powers 1, 2 and 3, are placed in a straight line at equal intervals

of 1 foot. Find where a small disc must be placed in order that it may be equally illuminated on both sides.

7. Two magnets, each of length 7·8 cm. and moment 156, are arranged to form a T, and a point is taken equidistant from the two. Find the force at this point in magnitude and direction for all possible arrangements of the poles of the magnets.

8. A circuit contains a battery of E.M.F. 2 volts, a resistance of 150 ohms (including that of the battery) and a galvanometer of resistance 56 ohms, shunted by a wire allowing only $\frac{1}{5}$ of the main current to pass through the galvanometer. Find the currents in the main circuit and in each branch.

9. The current from a battery of known resistance is sent through two wires of resistances R and S arranged in parallel. Find the condition that equal quantities of heat may be generated in the two wires in the same time.

10. In Colladon's experiment it was found that sound travelled through water at the rate of 1,451 metres per second. Find from this value the coefficient of elasticity of water at the temperature of the experiment.

Paper XXII

1. The bob of a simple pendulum of length 50 cm. weighs 10 gr. If it is displaced from its position of equilibrium until the string is horizontal and is then allowed to fall, find the tension in the string as the bob passes through its lowest point.

2. A faulty barometer reads 756 mm. and 755 mm. respectively when the atmospheric pressures are 760 mm. and 758·7 mm. What will be the reading when the pressure is 747 mm.?

3. Two spheres, of masses 100 and 200 kg. respectively, move towards each other with velocities of 1 and 2 metres per second. If the coefficient of restitution be 0·5, find how many calories of heat are generated by their collision, given that Joule's equivalent is 42×10^6 ergs per calorie.

4. The density of air at 0° C. and 760 mm. is 0·00129 gr. per c.c., and that of hydrogen is 0·00009 gr. per c.c. Two milligrammes of each are introduced into a vessel of capacity 30 c.c. What is the pressure inside the vessel at 20° C., assuming that zero on the absolute scale of temperature is −273° C.?

5. Two rays are incident normally on one face of an isosceles prism whose angle is $\sin^{-1}\frac{1}{\sqrt{3}}$ and refractive index 1·5. Prove that if one ray is refracted directly through the prism and the other emerges from the opposite face after being internally reflected at the base, the angle between the emergent rays is 60°.

6. An object is placed at a distance of 100·8 cm. from a convex lens, and the image is found to be at a distance of

23 cm. from the lens on the opposite side. Another lens is placed in contact with the first, and the image formed by the compound lens is 7·3 cm. farther off than in the first case. Find the focal lengths of the lenses.

7. A current deposits 0·65 gr. of copper in 10 minutes in a copper voltameter, and generates 46 calories of heat in a wire in the same time. If it be increased so as to deposit 0·1 gr. of copper per minute, how much heat is developed in the wire per hour?

8. A condenser A is formed of three parallel plates each of area 600 sq. cm. at distances of 3 mm. from each other in air, the two outer plates being connected and raised to a potential of 1,200 volts, while the inner plate is earthed. Another condenser has the same form, but its plates are three-fourths as large as the first and are separated by distances of 4 mm. from each other. Find the charge of the latter condenser when connected with A.

9. One arm AB of a Wheatstone's bridge contains a resistance of 8·1 ohms and also a coil whose resistance is 1·86 ohms. The arms AD, DC contain resistances of 20 ohms each, while BC contains a resistance of 10 ohms. Find the shunt which must be introduced between B and C in order that the bridge may be balanced for steady currents.

10. A tuning-fork of frequency 360 is sounded at one end of a glass tube, both ends of which are open. If the velocity of sound in air is 1,100 feet per second, find the length of the tube in order that it may act as a resonator, the radius of the tube being neglected.

Paper XXIII

1. Find the centre of pressure of a lamina $ABCD$ held in water with the point A in the surface and AC vertical, given that $AB = AD = 10$ cm. and $BD = BC = 15$ cm.

2. A body of mass 10 lb. moving with a velocity of 20 ft. per sec. strikes a second body. If the former moves on in the same direction with a velocity of 5 ft. per sec., and the latter begins to move with a velocity of 12 ft. per sec., find the mass of the second body.

3. Two grammes of air are admitted into a litre flask and heated to 50° C. Find the pressure inside the flask, given that the density of air at 0° C. and 760 mm. is ·001293 gr. per c.c. and that its coefficient of dilatation is 0·003665.

4. Find the velocity with which a piece of wax at 20° C. must be moving in order that its kinetic energy if converted into heat may be sufficient to melt it, given that the specific heat of wax is ·12, its latent heat 42, its melting-point 54° C., and that Joule's equivalent = 42,000,000 ergs per calorie.

5. Two sources of light, of candle-powers 3 and 4 respectively, are placed at a distance of 4 feet apart. Find the position in which a screen must be placed on the line joining them in order that it may receive equal illuminations from the two sources.

6. A convex lens produces a distinct image of a lamp on a screen at a distance of 50 cm. from the lamp. On moving the lens through a distance of 30 cm., a distinct image is again formed on the screen. Find the focal length of the lens.

7. The resistance of a lamp is 50 ohms, and the difference of potential between its terminals is 230 volts. Find the amount of heat generated in it per hour, assuming that the mechanical equivalent of heat is 42×10^6 ergs per calorie.

8. A Clark cell of E.M.F. 1·434 volts is in circuit with a galvanometer of resistance 5·5 ohms and a resistance of 100,000 ohms. The deflection observed is 67·5 scale divisions. A thermo-electric couple is substituted for the cell and the resistance altered to 50 ohms, when the deflection is found to be 91·5 divisions. Find the E.M.F. of the couple.

9. A cylindrical magnet of length 8·9 cm. and radius 0·4 cm. is executing vibrations in the earth's field. The mass of the magnet is 37·85 gr. and the time of 10 complete swings is 95·64 seconds. If the intensity of the earth's horizontal force is 0·18 dyne, find the moment of the magnet.

10. The velocity of sound in air at 0° C. and 760 mm. is 33,200 cm. per sec., while the density of air under the same conditions is 0·001293 gr. per c.c. *Find from these* data the value of the adiabatic coefficient of elasticity for air.

Paper XXIV

1. If a pendulum beats seconds at a place where g = 32·02 ft. per sec. per sec., find how much it will lose or gain per day at a place where g = 32·18.

2. An alloy, composed of metals of densities 7·8 and 8·8 gr. per c.c. respectively, has a volume of 200 c.c., and weighs 2,553 gr. Find the volume of each constituent.

3. Twenty grammes of water are cooled down very carefully to − 20° C. and then begin to freeze. If the latent heat of fusion of ice is 80, and the specific heat of water below zero is unity, find how much of the water will be converted into ice.

4. If the coefficient of linear expansion of steel is 0·000012, find what space must be left between 10-yard lengths of metal on a railway line so as to allow for expansion between the temperatures 0° C. and 40° C., assuming that the lengths are correct at 12° C.

5. The velocity of light in air is 186,000 miles a second. The vernier scale of a microscope focussed on a mark at the bottom of a beaker reads 3·54 cm.; water is poured into the beaker, and when the instrument is focussed on the mark and on the upper surface of the water the readings are respectively 4·15 cm. and 5·99 cm. Find the velocity of light in water.

6. A biprism is placed on the table of a spectrometer, and the readings of the images of the slit formed by reflection at the two faces are respectively 125° 26′ 30″ and 123° 58′. The readings of the two images formed by direct refraction through the two halves of the prism when the vertex is turned towards the collimator are respectively 203° 54′ and

203° 30′ 30″. Find the angle and refractive index of the prism.

Sin 44′ = 0·0128; sin 45′ = 0·0139; sin 23′ = 0·0067; sin 24′ = 0·0070.

7. A battery is in circuit with two resistances R_1 and R_2, whose sum is kept constant and equal to 8,000 ohms. There is also a tube of liquid in the circuit, connection with which is made by two copper plates. Two points in the liquid are connected to the quadrants of an electrometer, which then shows a deflection of 413 scale divisions. The quadrants are then connected to the ends of the resistance R_2, and it is found that when the latter is 2,800 ohms the deflection is 409 divisions. Find the resistance of the liquid between the given points.

8. The poles of a battery are connected successively to two wires of resistances 5 ohms and 4·2 ohms respectively, and it is found that the quantities of heat developed per second in the wires in the two cases are equal. Prove that the battery resistance is approximately 4·8 ohms.

9. Find the electric intensity at a point on the axis of a metal disc of radius 10 cm., charged with 50 units of positive electricity, the point being 8 cm. from the centre of the disc.

10. Two trains are approaching each other at the rate of 40 miles an hour. To an observer in one train the pitch of the whistle of the other appears to be that of the note A. If the frequency of the note C is 256 per second, and the velocity of sound in air is 1,100 feet per second, find the actual pitch of the whistle.

Paper XXV

1. The mass of a train is 200 tons, and the resistances due to friction, etc., are equivalent to 15 lb. per ton. Find the horse-power of an engine which draws the train at the rate of 50 miles an hour up an incline of 1 in 100.

2. A vessel contains water and mercury, the depth of each being 20 cm. Find the pressure on a square area of side 10 cm. held vertically with two edges parallel to the surface of the water, half the area being in each liquid (density of mercury = 13·6 gr. per c.c.).

3. 1 c.c. of air at 0° C. and 760 mm. weighs 0·001293 gr.; 1 c.c. of mercury under the same conditions weighs 13·596 gr., and the acceleration due to gravity is 981 c.g.s. units. Find how much work is done in heating 5·4 gr. of air under constant pressure from 15° C. to 33° C.

4. An engine consumes 1 cwt. of coal per hour; its efficiency is $\frac{1}{15}$, and the amount of heat generated by the combustion of 1 lb. of coal is enough to convert 15 lb. of water at 100° C. into steam at the same temperature. If the horse-power of the engine is 41·8, find the mechanical equivalent of heat (latent heat of steam = 536).

5. A convex lens is placed 14 cm. in front of a concave spherical mirror, and an object placed 20 cm. in front of the lens gives an image coincident with itself. If the focal length of the lens is 10 cm., prove that the radius of the mirror is 6 cm.

6. Find the dispersive power of a glass prism of angle 15° which gives a minimum deviation of 7° 48′ 40″ for red light, 7° 56′ for yellow light and 8° 15′ 20″ for violet light.

Sin 7° 30′ = 0·130526; sin 11° 28′ = 0·198798;
sin 11° 24′ = 0·197657; sin 11° 37′ = 0·201363;
25′ = 943; 38′ = 648.

7. The current from a battery of E.M.F. 2 volts and resistance 0·5 ohm is divided between three branches, containing respectively a 10-ohm coil, a 20-ohm coil and a galvanometer of resistance 5 ohms. Find the current through the galvanometer.

8. A condenser is formed of two thin spherical shells of brass of radii 10 cm. and 11 cm. respectively, the space between them being filled with equal thicknesses of vulcanite and glass of specific inductive capacities 2·5 and 7. Find the capacity of the condenser.

9. A magnet 7 cm. long is suspended in the earth's field of intensity 0·18 dyne. If the moment of the magnet is 85 units, find the couple required to deflect it through an angle of 60° from the magnetic meridian.

10. One metre of a given wire weighs 3·42 gr. Find the weight required to stretch a length 60 cm. of the wire in order that it may emit the note G when vibrating transversely in its fundamental mode, given that the frequency of the note E is 320.

Paper XXVI

1. Find the capillary depression of mercury in a glass tube of radius 0·2 mm., the surface tension of mercury being 520 c.g.s. units, its density 13·59 gr. per c.c. and its angle of contact with glass being 140° (cos 40° = 0·766).

2. A cylinder of metal of length 3·84 cm. and diameter 1·59 cm. weighs 66·70 gr. in air and 59·73 gr. in a given liquid. Find the density of the liquid.

3. A piece of copper is found to weigh 25·467 gr. in water at its temperature of maximum density. What will it weigh in water at 60° C. if its mean coefficient of expansion between these temperatures is 0·00005 and the density of water at 60° C. is 0·9834 gr. per c.c.?

4. From what height must a hailstone fall to the ground in order that it may just be melted, assuming that its temperature is always zero and that the mechanical equivalent of heat is 1,390 ft.-lb. per pound-degree C.?

Latent heat of ice = 80.

5. A luminous source and a screen are fixed on an optical bench at a distance of 40 cm. apart, and a lens is moved about on the bench until it gives a distinct image of the object on the screen. If the distance between the two positions of the lens is 10 cm. and the respective sizes of the images are 2 cm. and 1·5 cm., find the focal length of the lens and the size of the object.

6. The object glass and eyepiece of a microscope have focal lengths 1 cm. and 7 cm. respectively, and are separated by a distance of 15 cm. How far from the object glass must an object be placed so as to be in focus for a person who can see distinctly at a distance of 28 cm.?

7. A battery is in circuit with a galvanometer of resistance 5 ohms and a resistance of 20 ohms, and the deflection is noted. The resistance is then altered to 15 ohms, and a shunt of resistance 3 ohms connected to the terminals of the battery. If the deflection is the same as before, find the resistance of the battery.

8. A condenser is formed of thin conducting spherical shells of radii 10·5 cm. and 11 cm. respectively. The inner shell is earthed and the outer one raised to a potential of 10 electrostatic units. The latter is then connected to the outer shell of a similar condenser of radii 11·5 cm. and 12 cm. respectively, originally without charge, the inner shell of which is also earthed. Find the common potential after the connection is made and also the change in the energy of the system.

9. A magnet is placed on the arm of a magnetometer and its moment is calculated on the assumption that the force exerted on each pole of the suspended needle varies inversely as the cube of the distance of the centre of the magnet from the point of suspension. Prove that if the length of the magnet be about one-half of this distance, the percentage of error is about 14.

10. If the velocity of sound in air at 0° C. is 332 metres per second, and the coefficient of expansion of air is ·00366, find an expression for the velocity of sound at any temperature.

PAPER XXVII

1. A piece of wax weighing 3·27 gr. is attached to a piece of platinum weighing 15·61 gr. in air and 14·87 gr. in water. If the two together weigh 14·58 gr. in water, find the density of the wax.

2. Given that the time of oscillation of a simple pendulum depends only upon its length and the acceleration due to gravity, find by the principle of dimensions the law connecting these quantities. A pendulum oscillating at a place A makes 100 vibrations in 4 minutes; at another place B it makes 101 vibrations in the same time. Compare the values of g at the two places.

3. Thirty milligrammes of hydrogen, 200 mgr. of air and 100 mgr. of nitrogen are admitted into a half-litre flask at a temperature of 0° C. The flask is then heated to 50° C. Find the pressure inside it at this temperature.

Densities of hydrogen, air, and nitrogen, at 0° C. and 760 mm., are respectively ·00009, ·001293, and ·001260 gr. per c.c.; coefficient of expansion of each = ·003665.

4. Water flows at the rate of 10 gr. per sec. through a glass tube 1 metre long, of internal and external diameters 2 cm. and 2·2 cm. respectively, the outside of which is surrounded by steam at ordinary pressure. If the thermal conductivity of the glass is ·0014 c.g.s. unit and the temperature of the water entering the tube is 15° C., find its temperature as it leaves the tube at the other end.

5. A convex lens, the radii of whose surfaces are respectively 20 cm. and 12 cm., is placed upon an optical bench, and moved about until it gives an image of an object

at a minimum distance from the latter. If this distance is found to be 59·2 cm., find the focal length and refractive index of the lens.

6. Two circular discs, of radii 10 cm. and 15 cm., are placed coaxially with their planes parallel and at a distance of 25 cm. apart. Find where a luminous point must be placed on the axis in order that it may equally illuminate the two discs.

7. Three thin bar magnets, of length 10 cm. each, and moments respectively equal to 50, 80 and 100 units, are placed so as to form an equilateral triangle, with their opposite poles in contact. Find the force exerted by the combination at the centre of the triangle.

8. A battery is in circuit with a resistance of 900 ohms and a coil of resistance 1 ohm, to the ends of which a galvanometer is connected. If the resistance of the galvanometer is 640 ohms, find the amount of error introduced in the calculation of the current through the galvanometer by neglecting the resistance of the coil.

9. A coil of 50 turns, whose mean radius is 15 cm. and whose resistance is 10 ohms, rotates 20 times a second about a vertical diameter in the earth's field. If the intensity of the earth's horizontal force is 0·18 dyne and the coefficient of self-induction of the coil is 4×10^8 electromagnetic units, find the maximum current induced in the coil.

10. The velocity of sound in hydrogen at 0° C. and 760 mm. is 1,270 metres per second, and the density of hydrogen under the same conditions is 0·000088 gr. per c.c. Find the value of the adiabatic elasticity of hydrogen at the given temperature and pressure.

Paper XXVIII

1. A pump working at the rate of 20 horse-power is employed to empty a cylindrical well of water of radius 2 feet and height 200 feet. If the well is closed at the bottom, and the water originally stands in it to a height of 180 feet, find how long it will be before the well is empty.

2. A mass of 10 lb., moving with a velocity of 20 ft. per sec., overtakes a mass of 5 lb. moving with a velocity of 15 ft. per sec. in the same direction. If the coefficient of restitution between the masses is $\frac{1}{2}$, find the loss of kinetic energy after impact.

3. A compensation pendulum is made of bars of iron and brass. If the coefficients of linear expansion of these metals are respectively 0·000012 and 0·000019, and the total length of the brass rods is 36·4 cm., find how long the iron rods should be.

4. A glass bottle holds 20·339 gr. of mercury at 0° C. and 20·023 gr. at 100° C. If the mean coefficient of cubical expansion of mercury between these temperatures is 0·000182, find the coefficient of cubical expansion of glass.

5. A ray of light is refracted through a sphere in such a manner that it passes through the extremities of two radii which make an angle a with each other. Prove that, if D is the deviation of the ray, $\cos \frac{a - D}{2} = \mu \cos \frac{a}{2}$.

6. A luminous point is placed 16 cm. in front of a convex lens of focal length 10 cm. Behind this lens, at a distance of 8·6 cm. from it, is placed another convex lens of focal length 12 cm. Find where a third lens must be placed

in order that a small pencil of rays from the object may emerge as a parallel beam after refraction through the three lenses.

7. A triangle *ABC* is formed of wires, the resistance of each side being 3 ohms. A point *O* inside the triangle is joined to the three vertices by wires each of resistance 2 ohms, and a current enters at one vertex and leaves at *O*. Find the equivalent resistance of the network.

8. A quadrant electrometer when charged in the usual way by a Clark cell shows a deflection of 77 scale divisions. One plate of a condenser is connected to the positive terminal of the cell, while the negative terminal is earthed, and the other plate is connected to the insulated quadrants. The deflection observed is 71. If the quadrants of the electrometer have a capacity of 40 electrostatic units, find the capacity of the condenser.

9. A magnet of moment 250 is suspended in the magnetic meridian, and is then turned until it makes an angle of 60° with its former direction. If the intensity of the earth's horizontal force is 0·18 dyne, find the change in the potential energy of the magnet.

10. A whistle emits a note of frequency 300. What will be the frequency of the note heard by an observer travelling towards it at the rate of 60 miles an hour?

Velocity of sound = 1,100 feet per second.

Paper XXIX

1. A cylinder 75 cm. high and 10 cm. radius stands in an upright position and is filled with water. If it is now tilted until its axis makes an angle of 30° with the vertical, find the magnitude of the resultant pressure on the base, the top of the cylinder being open.

2. A sphere of mass 2 kg. and radius 10 cm. is rolling along the ground with a velocity of 10 cm. per sec. Find what force will be needed in order to bring it to rest after it has travelled 10 metres.

3. A quantity of air has a volume of 2 litres at 50° C. and 780 mm., while another quantity occupies a volume of 4 litres at 20° C. and 740 mm. If the two are admitted into a 5-litre vessel at 15° C., find the pressure they will exert, given that the coefficient of expansion of air is 0·00366.

4. How many gramme-degrees of heat will flow per minute through each square metre of a plate of metal 20 cm. thick, the two faces of which are kept at 10° C. and 150° C. respectively, the thermal conductivity of the metal being 0·18 c.g.s. unit?

5. An achromatic combination of focal length 2 feet has to be constructed of two lenses whose refractive indices are 1·51 and 1·63, and whose dispersive powers are in the ratio 12 : 17. Find the focal lengths of the two lenses required.

6. Two prisms, each of angle A, and of refractive indices μ_1 and μ_2, are placed with one face in contact, and their refracting angles turned in the same direction. If a ray of light be incident upon the face of one of them at right angles,

find the condition that it may emerge from the other in a direction perpendicular to its original direction.

7. A wire of resistance 15 ohms forms part of a circuit containing a battery, the total resistance of the circuit being 25 ohms. Find the shunt required to be placed across the ends of this wire in order that the heat developed in it may be only half as much as it is.

8. A magnet makes 10 oscillations in 3 min. 6 sec. at a place where the dip is 67°, and 10 oscillations in 3 min. 12 sec. at another place where the dip is 72°. Compare the total magnetic forces at the two places (sin 18° = 0·309; sin 23° = 0·391).

9. An insulated sphere of radius 10 cm., charged with 300 electrostatic units of electricity, is placed at a great distance from an uncharged insulated sphere of radius 6 cm. If the two are joined by a wire, find the extra charge which must be given to the former in order to maintain its potential constant, and find the changes that occur in the energy of the spheres.

10. A closed tube 3 feet long resounds to the lowest note given by an open tube of length 6 feet and diameter 10 inches. Find the diameter of the first tube.

Paper XXX

1. A small balloon weighing 10 gr. when empty is inflated with coal gas. Find its volume when it just begins to rise, given that the masses of 1 litre of air and of coal gas are respectively 1·29 gr. and 0·53 gr.

2. Find the total pressure on a rhombus $ABCD$, of side 2 feet, held in water with one diagonal vertical and one vertex in the surface, the angle at that vertex being 60°.

3. The density of aqueous vapour is five-eighths of that of dry air at the same temperature and pressure. Find the weight of a mixture consisting of a litre each of nitrogen and oxygen saturated with water vapour at 15° C. and 755 mm., the pressure of the vapour at this temperature being 12·7 mm., the density of air at 0° C. and 760 mm. being 0·001293 gr. per c.c., 1 litre of hydrogen weighing 0·0896 gr. under the same conditions, and the atomic weights of oxygen and nitrogen being 16 and 14 respectively.

4. A long capillary tube of glass contains a thread of mercury at 10° C., the length of which according to a brass scale attached to the tube is found to be 150·56 cm. What will be the length recorded by the scale at a temperature of 55° C.?

Coefficient of cubical expansion of glass is 0·000026, of mercury 0·00018, and of brass 0·000052.

5. A plane mirror is placed at a distance of 15 inches from a convex lens, the mirror being perpendicular to the axis of the lens. An eye placed at a distance of 8 inches from the lens on the side remote from the mirror sees an image of itself by parallel rays. Prove that the focal length of the lens is 6 inches.

6. The refractive index of a prism of angle 60° for light of mean refrangibility is 1·627. The minimum deviation for red light is 48°, and that for blue light is 50° 36′. Find the refractive indices for red and blue light, and the dispersive power of the prism (sin 54° = 0·809; sin 55° 18′ = 0·822).

7. A battery of E.M.F. 2·2 volts and resistance 3 ohms is in circuit with a coil of resistance 18 ohms and a galvanometer of resistance 5 ohms, shunted by a wire of resistance 3 ohms. Find the difference of potential between the terminals of the coil.

8. Two short bar magnets are placed in the magnetic meridian north and south of a small suspended needle, with their centres distant 30 cm. and 20 cm. from it respectively. If the needle is not deflected, compare the moments of the magnets.

9. Find the capacity of 24 Leyden jars joined up in six parallel groups of four each, the area of the metal covering of each being 250 sq. cm. and the dielectric being formed of material 5 mm. thick and of specific inductive capacity 2·5.

10. Four strings of the same weight and material are stretched with the same force, and when vibrating give the notes A, C, E′ and A′. Find the ratio of their lengths.

Paper XXXI

1. A body weighs 160 gr. in air, 145 gr. in one liquid and 125 gr. in another liquid. How much will it weigh in a mixture of the liquids in the proportion of 3 : 2 by volume?

2. A weight W is attached to a vertical elastic string and is allowed to fall vertically from the position in which the string is just not stretched. Prove that when the weight next comes to rest the tension in the string is $2W$.

3. A gas is saturated with vapour and exerts a pressure on its containing vessel equal to that of 100 cm. of mercury. The gas is compressed until it occupies one-quarter of its original volume, and the pressure is then found to be that due to 380 cm. of mercury. Find the pressure of the vapour in the first instance.

4. Find how much work has to be spent in compressing 1 cubic metre of air at the normal pressure of 76 cm. into a vessel of capacity 50 litres, the density of mercury being 13·6 gr. per c.c., and the acceleration due to gravity being 981 cm. per sec. per sec.

5. An object 2 inches high is placed on the axis of a hemisphere of glass of radius 4 inches and refractive index 1·5. If the object be 6 inches in front of the curved surface, find the position and size of the image.

6. A ray of light enters one face of a prism of angle 60° at grazing incidence, and after refraction through the prism it emerges in a direction making an angle of 45° with the other face. Find the refractive index of the material.

7. Three cells of E.M.F. 1·1, 1·4 and 1·9 volts respectively, and of internal resistances 1·8, 0·5 and 1·2 ohms,

are joined in parallel. What current will they send through an external resistance of 5 ohms?

8. Find the coefficient of self-induction of a solenoid 50 cm. long, of mean radius 5 mm. and having 30 turns of wire per cm. of length.

9. A small pith ball weighing ·05 gr. is suspended by a silk thread 30 cm. long, being charged with 5 electrostatic units of positive electricity. Another small charged body is fixed 2 cm. away from it, and the first ball is repelled until the string makes an angle of 5° with the vertical. Find the charge on the small body, given that sin 5° = 0·0872 and g = 981 cm. per sec. per sec.

10. A string is stretched by hanging on a weight of 50 kg., and it is found that a small wave travels along it with a velocity of 10 metres per second. If the stretching weight be increased to 90 kg., find the velocity of propagation of the wave.

Paper XXXII

1. Find the value of the mechanical equivalent of heat in foot-pound-second units on the Fahrenheit scale, given that in c.g.s. units on the Centigrade scale it is $4{\cdot}2 \times 10^7$ ergs, that 1 ft. = 30·5 cm. and that 1 lb. = 453·6 gr.

2. A particle is projected along the inside of a smooth vertical circle from the lowest point. If the circle is of radius 4 feet, and after leaving the circle the particle passes through the centre, find the velocity of projection.

3. A piece of metal weighs 133·350 gr. in vacuo, 118·458 gr. in water at 15° C., and 118·501 gr. in water at 50° C. If the coefficient of cubical expansion of the metal is 0·00005, find the mean coefficient of expansion of water between the two temperatures.

4. A body weighing 50 gr. is dropped into a Bunsen's ice calorimeter. The initial temperature of the body is 97° C., and the movement of the thread of mercury indicates a contraction of 0·523 c.c. in the mixture of ice and water. The latent heat of fusion of ice is 80, and the density of ice at 0° C. is 0·917 gr. per c.c. Find the specific heat of the body.

5. An eye is placed close to a sphere of glass which is silvered at the back. Find the magnification of the image which the eye sees of itself if the refractive index of the glass is 1·6.

6. Two plane mirrors are inclined at an angle of 45°, and a luminous point is placed between them. Show that seven images of this point can be seen by an eye placed between the mirrors, and draw a pencil of rays showing how an image is seen by two reflections at each mirror.

7. A current divides between three wires, of copper, iron and lead respectively, arranged in parallel. The diameter of the iron wire is twice that of the copper, and its specific resistance is six times as great; the diameter of the lead wire is twice that of the iron, and its specific resistance twice as great. Compare the lengths of the wires if the same amount of heat is developed in each.

8. A battery of E.M.F. 4 volts and of negligible resistance is in circuit with a resistance of 100,000 ohms and also a galvanometer of resistance 825 ohms. The deflection observed is 5°. The current from the battery is then sent through an unknown resistance and a resistance of 1 ohm in series, to the ends of which the galvanometer is connected. In order to obtain the same deflection as before, a resistance of 654 ohms has to be placed in the galvanometer circuit. Find the value of the unknown resistance.

9. A condenser is formed of concentric spherical shells of radii 30 cm. and 32 cm., the space between which is filled with material of specific inductive capacity 7. The inner sphere is connected to earth, and the outer is raised to a potential of 50 electrostatic units. If the condenser be discharged, find how much heat will be produced, assuming that all the energy of the discharge is converted into heat, and that Joule's equivalent is 42 million ergs per calorie.

10. A string, of length 50 cm., weighing 0·015 gr., emits a note of frequency 512 when bowed. Find the value of the stretching force.

Paper XXXIII

1. A body weighs 48·976 gr. in air and 27·655 gr. in water of density 0·997 gr. per c.c. If the density of air is 0·00129 gr. per c.c. and that of the weights employed is 7·8 gr. per c.c., find the density of the body.

2. In an Atwood's machine the two masses are each 100 gr., and the mass of the rider is 20 gr. The mass and rider leave the platform, and the rider is caught by the ring 1 second later: if in the next 2 seconds the space passed through by the masses is 179 cm., find the value of g.

3. The latent heat of ice on the c.g.s. Centigrade scale is 80, and the mechanical equivalent of heat is $4{\cdot}15 \times 10^7$ ergs. What will be the values of these quantities reckoned on the Fahrenheit scale?

4. Two bars of metal are coated with wax and supported horizontally with one end in a vessel of boiling water. The diameters of the bars are respectively 1 cm. and 1·5 cm., and after some time it is found that the distances along the bars to which the wax has melted are respectively 15 cm. and 18 cm. Compare the thermal conductivities of the metals.

5. Find the points of unit magnification of a double convex lens of refractive index 1·52, the radii of whose surfaces are 25 cm. and 32 cm. respectively, the lens being 6 mm. thick at the centre.

6. Six equal prisms are placed together with their edges in contact. A ray of light passes through them so that the deviation produced by each is a minimum, and emerges in

a direction parallel to its original direction. If the angle of each prism is $2i$, and the refractive index is 1·5, prove that $\tan i = \frac{3 + \sqrt{3}}{6}$.

7. A calorimeter of water equivalent 3·5 gr. contains 45·1 gr. of water at 15° C. A current is passed through a wire immersed in the water, and after 10 minutes the temperature is found to have risen to 25° C., the deflection of a galvanometer in the circuit being 18°. The galvanometer has 2 turns of diameter 15·5 cm. Calculate the resistance of the wire, given that $H = 0{\cdot}18$ dyne, $\tan 18° = 0{\cdot}325$, $J = 42 \times 10^6$ ergs per calorie.

8. A pendulum consists of a sphere of mass 5 gr. suspended by a silk thread, and the time of 10 small oscillations is 10·1 seconds. The sphere is charged with unit quantity of electricity, and another charged sphere is placed vertically beneath it at a distance of 10 cm. Prove that the time of a small oscillation will now be 1 second if the charge on the second sphere is approximately $10g$ units.

9. A magnet, 7·5 cm. long and of moment 64, is suspended by a thread in the earth's field of intensity 0·19 dyne. Find the magnitude of the couple required to deflect it through an angle of 60° from the magnetic meridian.

10. A string attached to one of the prongs of a tuning-fork passes over a smooth pulley and carries a mass of 30 gr. at its other end. When the fork is vibrating it is found that the string divides into 6 loops. Find what mass must be attached to the end of the string in order to make it divide into 7 loops.

Paper XXXIV

1. A ladder 40 feet long, weighing 30 lb., rests with one end on the ground and the other against a vertical wall, the coefficient of friction between the ladder and each being $\frac{1}{3}$. If the ladder is inclined to the horizon at an angle of 60°, find how far a man weighing 150 lb. can ascend along it.

2. Find the volume of platinum of density 21·2 gr. per c.c. which must be attached to a sphere of cork of radius 5 cm. and of density 0·25 gr. per c.c., in order that it may just float in a liquid of density 0·89 gr. per c.c.

3. The critical temperature of a gas obeying Van der Waal's equation $\left(p + \frac{a}{v^2}\right)\left(v - b\right) = Rt$, is 31° C., and the critical pressure is 76 megadynes per sq. cm. Prove that $R = 2b \times 10^6$.

4. A thousand cubic feet of air at 775 mm. and 71° F. weigh 78·43 lb.; find the weight of a litre of air at 0° C. and 760 mm., given that 1 metre = 39·37 inches and that 1 kg. = 2·2 lb.

5. A concave spherical mirror is placed on a table, and the image of a pin held at a height of 22·7 cm. above its centre is found to coincide with the pin itself. Water is poured into the mirror to a depth of 1·4 cm. at the middle point, and in order that the object and image may again coincide the pin has to be moved through a distance of 5·3 cm. Find the refractive index of the water.

6. How would you arrange two convex lenses of focal lengths 12 cm. and 20 cm. so as to be equivalent to a

single lens of focal length 15 cm.? Find also the position in which an object must be placed in order to give an image twice the size.

7. The current from a battery is sent through a copper voltameter, and 0·14 gr. of copper is deposited in 400 seconds, the resistance of the circuit being 5 ohms. Two points in the circuit, the resistance between which is 1 ohm, are joined by a wire of unknown resistance, and also connected to a galvanometer of resistance 4 ohms. If the current which flows through the galvanometer is 0·65 ampere, find the value of the unknown resistance (electro-chemical equivalent of copper = 0·000328 gr. per coulomb).

8. A quadrant electrometer, when charged in the usual way by a Clark cell, gives a deflection of 48 scale divisions. The insulated terminal of the cell is connected to the inner plate of a cylindrical condenser, the outer plate of which is in connection with the insulated quadrants, and the deflection observed is 32 divisions. The length of the condenser is 50 cm., its internal and external radii are 2 cm. and 2·5 cm. respectively, and the dielectric is air. Find the capacity of the quadrants of the electrometer, given that exp. log. 1·25 = 0·446.

9. A tram is travelling at the rate of 4 metres per second along insulated rails 1·25 metres apart. Find the E.M.F. between the ends of the rails, given that the earth's horizontal intensity is 0·18 dyne, and that the tangent of the angle of dip is 2·5.

10. If the velocity of sound in air is 332 metres per second, find the length of the shortest closed tube which will resound to a tuning-fork of frequency 256.

Paper XXXV

1. The receiver of a compression pump has a volume of 2 litres, and the volume of the barrel is 100 c.c. If the initial pressure inside the receiver is 760 mm., what will be the pressure after 20 strokes?

2. A particle is projected from the lowest point along a smooth tube in the form of a vertical circle of radius 5 feet. If on passing the highest point the particle exerts no pressure on the tube, find the velocity of projection.

3. A perfect gas expands isothermally. If its original volume is 500 c.c., and the pressure changes from 750 mm. to 850 mm., find the amount of work done in expansion. Density of mercury = 13·596 gr. per c.c.; g = 981 cm. per sec. per sec.

4. In order to keep an engine working at the rate of 3,000 horse-power, 4 tons of coal have to be consumed every hour. Assuming that the mechanical equivalent of heat is 1,390 foot-pounds per pound-degree Centigrade, and that the combustion of 1 lb. of coal generates sufficient heat to melt 1 cwt. of ice, find the efficiency of the engine (latent heat of fusion of ice = 80).

5. A luminous object is placed on an optical bench, and an image is formed by a convex lens on a screen 60 cm. away. The object and screen are kept in their places, while the lens is shifted through a distance of 26·8 cm., when the image on the screen is again in focus. Find the focal length of the lens.

6. The refractive indices of a material for red and violet rays are 1·627 and 1·662 respectively; find its dispersive power. A lens is formed of this material, and another is

formed of a material for which the corresponding indices are 1·527 and 1·543. Find the focal lengths of the lenses in order that they may form an achromatic combination of focal length 20 cm.

7. The current from a battery of cells arranged in q parallel rows is sent through an external resistance R. The cells and connecting wires are then arranged in series, and the current is sent through a resistance S. Prove that, if the battery resistances be neglected, the quantities of heat developed per second in the two cases are equal if $S = q^2R$.

8. Two long thin needles, equal and equally magnetised, are suspended from the same point by threads 25 cm. long, and set parallel to each other so that the angle between each thread and the vertical is 1°. If each needle is 20 cm. long and weighs 10 gr., find its magnetic moment.

9. A Leyden jar is charged to a potential of 1,000 volts. It is then connected with another jar whose area is twice as large and whose dielectric is half as thick, while the specific inductive capacities of the dielectrics are as 5 to 14. Find the common potential and the charges of the jars.

10. A train travelling at the rate of 60 miles an hour blows a whistle, the pitch of which is that of the note G. What will be the pitch of the note heard by an observer in a train travelling towards the first at the rate of 50 miles an hour? (Frequency of note G = 384; velocity of sound in air = 1,100 feet per second.)

Paper XXXVI

1. If the numerical value of the acceleration due to gravity be 32 when the units of length and time are the foot and the second, find its value when the units are 1 metre and 3 minutes respectively (1 metre = 39·37 inches).

2. Water stands to the heights of 30 feet and 12 feet on the two sides of a dock-gate rectangular in shape and 20 feet broad. Find the magnitude of the resultant pressure on the gate and determine its point of application.

3. In finding the specific heat of a metal it is found that on immersing 50 gr. of metal at 95° C. in a calorimeter containing 80 gr. of water at 15° C. the temperature rises to 20° C. If the water equivalent of the calorimeter is 12 gr., find approximately the greatest error that can occur in the result if each thermometer reading be liable to an error of half a degree.

4. A large flask furnished with a stop-cock contains air at a pressure of 15 mm. The stop-cock is suddenly opened and then quickly closed again, and the pressure, after the flask has had time to cool down, is found to be 198·3 mm. If the external pressure is 758 mm., find the ratio of the specific heats of air.

5. A reflector is elliptical in shape, and a luminous point is placed at its centre. Prove that a small pencil of rays incident on it can only proceed as a parallel pencil after reflection if the eccentricity is greater than $\sqrt{2}$.

6. A ray of light passes through a prism of refractive index $\sqrt{3}$ in such a way that the angle of incidence is twice the angle of emergence and the latter is equal to the angle of the prism. Prove that the angle of the prism is 60°.

ABCD are the four corners of a Wheatstone's bridge. The arms *AB*, *BC*, *CD*, *DA* contain resistances of 10, 100, 1,587 and 158·5 ohms respectively. A battery of 1·45 volts is connected to *A* and *C*, and a galvanometer of resistance 220 ohms is connected to *B* and *D*. If the observed deflection on the scale of the galvanometer is 4 mm., find the current corresponding to a deflection of 1 mm.

8. A thin hoop, whose diameter is 1 metre, is held in a vertical position at right angles to the magnetic meridian, and is then allowed to fall over on the ground. Find in coulombs the quantity of electricity which passes round the hoop, given that the earth's horizontal force is 0·18 dyne, and that the tangent of the angle of dip is 2·39.

9. An ordinary accumulator will give a current of 1 ampere for 24 hours. The chemical change that takes place in the cell is the reduction of lead peroxide to lead monoxide. Find the least amount of lead peroxide necessary on one of the plates, given that the electro-chemical equivalent of copper is 0·000328 gr. per coulomb, and that the atomic weights of oxygen, copper and lead are respectively 16, 63·2 and 206·4.

10. An open pipe gives a note of frequency 256 when the temperature of the air is 15° C. What will be the frequency of the note emitted at 87° C., assuming that the mode of vibration is the same, and neglecting the expansion of the tube?

Paper XXXVII

1. A body moving northwards with a velocity of 30 ft. per sec. meets another body of twice its mass moving eastwards with a velocity of 20 ft. per sec. If the two bodies coalesce, find the direction and magnitude of the resultant velocity.

2. A test-tube of length 15 cm. is plunged mouth downwards into a vessel containing mercury. How deep must it be plunged in order that the liquid may rise one-third of the way up the tube? (height of barometer = 75·8 cm.).

3. By how much will the mass of air in a room 30 feet long, 20 feet broad and 15 feet high change when the temperature rises from 50° F. to 65° F., the pressure remaining constant at 755 mm.? The mass of a cubic foot of water = 62·3 lb., and the specific gravity of air at 32° F. and 760 mm. is 0·001294.

4. The product of the pressure and specific volume of a substance is proportional to some power of the absolute temperature. Prove that the latent heat of dilatation varies directly as the pressure.

5. A bright point is situated on a curve, and the illumination at any point of the curve is proportional to its distance from the luminous point. Prove that the curve is a circle.

6. A ray of light is incident at a very small angle θ on a prism of angle A and refractive index μ. If the deviation produced is D, prove that, approximately,

$$\theta \sin\left(A + \frac{D}{2}\right) = \frac{\mu - 1}{2} \sin A \operatorname{cosec} \frac{D}{2} - \cos\left(A + \frac{D}{2}\right).$$

ABC is a triangle of wire, the resistances of *AB*, *AC* and *BC* being 2, 3 and 3 ohms respectively. A fourth wire *AD*, passing round a galvanometer of resistance 5 ohms, can make sliding contact with *BC*. A current enters at *A* and leaves at *D*. Prove that the currents through the galvanometer in the two cases where *D* is situated at the points of trisection of *BC* are in the ratio of 21 to 22.

8. A company supplies electrical energy at the rate of 6d. per horse-power-hour. Find how much it will cost to keep up 40 arc lights for 6 hours, given that each light takes 12 amperes at a pressure of 160 volts, and that 1 horse-power = 746 watts.

9. Four equal bar magnets are placed with their north poles in contact so that the angle between adjacent magnets is 90°. Find the direction in which a free unit north pole would move if placed at a point midway between the south poles of two of the magnets. Find also the force on this pole if each magnet is 20 cm. in length, and has a moment of 560 units.

10. Two metres of a given wire weigh 5 gr. Find the note emitted by a 60 cm. length of the wire when stretched by a weight of 30 kg., and vibrating transversely in its fundamental mode.

Paper XXXVIII

1. When a mass of 50 gr. is suspended from a piece of elastic the length of the latter is 64 cm.; with an additional 10 gr. the length is 67 cm. Find the mass of a body which stretches the elastic to a length of 72 cm.

2. A cube is filled to a depth of 15 cm. with a liquid whose density is 1·4 gr. per c.c., and a second liquid is poured on top of the first until the pressures on the two parts of one side of the vessel in contact with the liquids are equal. If the density of the second liquid is 0·95 gr. per c.c., find its depth.

3. A weight thermometer weighs 8·162 gr. when empty, 89·516 gr. when filled with mercury at 0° C. and 89·053 gr. when filled with mercury at 100° C. If the coefficient of expansion of the material of the thermometer be 0·000027, find the coefficient of absolute expansion of mercury.

4. The specific heat of air at constant volume is 0·169, and its specific gravity is 0·001293 at 0° C. and 760 mm. Find how much heat is required to warm a room 20 feet long, 25 feet broad and 15 feet high from 5° C. to 15° C., given that 1 cubic foot of water weighs 28·35 kg., and assuming that the change is isometric.

5. Two thin convex lenses, made of the same material, are of focal lengths 12 cm. and 18 cm. respectively. Find the focal length of the equivalent lens when they are separated by a distance of 15 cm.

6. An astronomical telescope, having an eyepiece of focal length 1 inch, is in adjustment for a person who can see most distinctly at a distance of 9 inches. Prove that, in

order that it may be in adjustment for a person whose distance of distinct vision is 14 inches, the eyepiece must be moved through $\frac{1}{30}$ of an inch.

7. Two circular discs, of diameter 1 foot each, are placed parallel to each other at a distance of $\frac{1}{4}$ inch apart. If one of them is earthed and the other is charged to a potential of 10,000 volts, prove that the force of attraction between them is roughly equal to the weight of 2 oz. (1 kg. = 2·2 lb.; 1 ft. = 30 cm.).

8. The terminals of a battery are connected to the ends of a solenoid having n turns per unit length, and the circuit also includes an unknown resistance R. The ends of the latter are connected, through a galvanometer, to two brushes, placed respectively at the centre and circumference of a wheel of radius a revolving with angular velocity ω inside the solenoid, the axis of rotation being the axis of the solenoid. If there is no current through the galvanometer, prove that $R = 2\pi n\omega a^2$.

9. A circuit contains a secondary cell of E.M.F. 2 volts, a resistance of 500 ohms, a 1-ohm coil, and an unknown resistance. A galvanometer is connected to the ends of the coil, and the deflection observed is 26 scale divisions when the resistance of the galvanometer circuit is 720 ohms. The galvanometer is then connected to the ends of the unknown resistance, and the deflection observed is 18 scale divisions when the circuit has a resistance of 250 ohms. Find the value of the unknown resistance, and the difference of potential between its extremities.

10. A closed tube 130 cm. long gives a certain note when full of oxygen. What must be the length of a closed tube full of air which will give the same note, if the densities of oxygen and air are respectively 0·00143 and 0·00129 gr. per c.c.?

Paper XXXIX

1. A body weighs 95 gr. in air, 81·5 gr. in one liquid and 79·8 gr. in another liquid. If it weighs exactly 80 gr. in a mixture of the two, find in what proportions the liquids are mixed.

2. A barometer tube 85 cm. long contains a small quantity of air above the mercury, so that it reads 75 cm. when the true atmospheric pressure is 76 cm. What will be the true pressure when the barometer reads 74·5 cm?

3. Three parts of water at 0° C. are mixed with one part of water at 80° C., and the resulting temperature is 20·13° C. Equal parts of water at 0° C. and 40° C., when mixed, give a resulting temperature 20·03° C. Assuming that the mean specific heat of water between 0° and t° C. can be expressed in the form $1 + at + bt^2$, find the values of a and b from the experiment.

4. A glass weight thermometer contains 25 gr. of a solid of density 8·8 gr. per c.c., and also 154 gr. of mercury of density 13·596 gr. per c.c. at 0° C. The thermometer is heated to 100° C., and 2·421 gr. of mercury are expelled. Find the coefficient of cubical expansion of the solid, given that the coefficients for mercury and glass are respectively 0·00018 and 0·000026.

5. A ray passes through a prism of angle A and refractive index μ. If the angles of incidence and emergence are B and C, prove that

$$(\mu^2 - 1)\sin^2 A = (\cos A - \cos B + C)(\cos A + \cos B - C).$$

6. Find the thickness and refractive index of a convex lens of diameter 5·1 cm. and focal length 16·85 cm., the radii of whose surfaces are 20·9 cm. and 15·7 cm.

7. A calorimeter of water equivalent 5 gr. contains 85 gr. of water initially at 15° C. A current is passed through a coil of wire immersed in the water and also through a tangent galvanometer having 20 turns of wire of mean radius 6 cm. The mean deflection of the galvanometer is 41°, and the temperature of the water rises to 33° C. in 10 minutes. Find the resistance of the coil ($J = 4{\cdot}2 \times 10^7$ ergs; tan 41° = 0·87).

8. A framework $ABCD$ consists of wires AB, BD, DC, CA of resistances 2, 4, 3 and 6 ohms respectively, and B, C are joined by a wire of resistance 5 ohms. A current of 0·8 ampere enters at A and leaves at D. Find the magnitude of the current in BC and the difference of potential between C and D.

9. Two insulated spheres of radii 4 cm. and 6 cm. respectively are charged to potentials 200 and 400 volts. They are then connected by a wire. A point on this wire is then connected by a second wire to another charged sphere of radius 10 cm. at a potential of 1,000 volts. Neglecting the capacities of the wires, find the quantities of electricity which pass along them in the two cases.

10. The velocity of sound in carbon dioxide at 0° C. is 262 metres per second. Find the ratio of the specific heats for the gas, given that at 0° C. and 1·013226 megadynes pressure the density of air is 0·001293 gr. per c.c., and that the density of carbon dioxide is to that of air as 20 : 13.

Paper XL

1. A globe is formed of two hemispherical shells of radius 50 cm. fitting closely together. The globe is exhausted of three-fourths of its air. Find the force required to separate the shells, the atmospheric pressure being 1 megadyne per sq. cm.

2. A regular tetrahedron of side 50 cm. is immersed in water with its base horizontal and its vertex 30 cm. below the surface. If the height of the water barometer is 10·3 metres, find the pressures on the base and on one of the faces.

3. Find the weight of 10 litres of oxygen saturated with water vapour at 50° C. and 785 mm., given that 1 litre of hydrogen at 0° C. and 760 mm. weighs 0·896 gr., that the densities of oxygen and water vapour with respect to hydrogen are respectively 16 and 8·9, and that the pressure of water vapour at 50° C. is 92 mm.

4. The coefficients of cubical expansion of mercury and brass are respectively 0·00018 and 0·000054. The reading of a barometer on a brass scale at 18° C. is 746·5 mm. Find the corrected reading at 0° C.

5. The difference in the readings of a spherometer when placed on a plane surface and on a concave spherical mirror is 0·0285 cm., and the feet of the instrument form an equilateral triangle of side 3 cm. A luminous point is placed 70 cm. in front of the mirror. Find the position and the magnification of the image.

6. An object is placed 6 cm. in front of a convex lens of focal length 4·5 cm. Find where a concave spherical

mirror of radius 14 cm. must be placed behind the lens so as to give an image twice as large as the object. What is the nature of the image?

7. A cylindrical bar magnet, of length 10 cm. and radius 0·6 cm., is oscillating in the earth's field of horizontal intensity 0·19 dyne. If the mass of the magnet is 67·42 gr. and the time of 20 complete vibrations is 3 min. 53 sec., find its moment.

8. A circuit is formed of a battery of E.M.F. 2·5 volts and resistance 1·4 ohms, and 3 wires of resistances 3, 5 and 7 ohms respectively, arranged in series. The ends of the middle wire are joined by wires of resistances 4 ohms and 6 ohms. Find the difference of potential between the ends of these, and also the currents that flow through them.

9. A Faraday's disc revolves 10 times a second in a field of strength 2,000. If the radius of the disc is 10 cm. and the resistance of the circuit is 1·25 ohms, find the strength of the induced current.

10. A rod of wood 1 metre long floats vertically in water with 90 cm. of its length immersed. When allowed to vibrate longitudinally the lowest note emitted has a frequency of 320 per second. Find the value of Young's modulus for the wood.

Paper XLI

1. A column of mercury 50 cm. in length is contained in a U-tube open at both ends. Prove that the time of a vertical vibration when the column is slightly displaced from its position of equilibrium is about 1·003 seconds.

2. A reservoir in the form of a square of 200 yards side contains water to the depth of 20 feet. The water is used to drive an engine at the level of the bottom of the reservoir. If the reservoir can be emptied in 24 hours, find the average horse-power maintained in the engine, given that 1 cubic foot of water weighs 62·3 lb., and that 1 horse-power = 33,000 foot-pounds per minute.

3. A hollow cubical vessel, formed of plates of metal 20 cm. square and 5 mm. thick, is filled with ice and placed inside a tank of water at 50° C. If the thermal conductivity of the metal is 0·18 c.g.s. unit, find how long it will be before all the ice is melted.

4. Find the weight of a litre of air saturated with water vapour at 20° C. and subject to a pressure of 74·36 cm. of mercury, the density of dry air at 0° C. and 76 cm. being 0·001293 gr. per c.c., and the pressure of water vapour at 20° C. being 17·4 mm.

5. The light from a slit is viewed by an eyepiece at a distance of c cm. from it, through a biprism placed between them at a distance of a cm. from the slit. If λ is the wave length of the light used, δ the angle of the prism, and x the mean width of the interference bands observed, prove that $\tan \delta = \frac{c\lambda}{2ax}$ approximately. If the biprism is 50 cm.

from the slit and the eyepiece, the angle of the prism is 22′ 50″, and the mean width of a band is ·0886 mm., find the wave length of the light used, given that tan 22′ 50″ = 0·00666.

6. Two convex lenses of focal lengths 3 inches and 4 inches respectively are placed at a distance of 6 inches apart, and a luminous object 1 inch high is situated on the common axis of the lenses at a distance of 4 inches in front of the lens of greater power. Find the position and size of the image.

7. Six wires are joined to form a tetrahedron $ABCD$. The resistances of AB, AC and AD are each 1 ohm, and those of BC, BD and CD are respectively 2, 3 and 4 ohms. A current enters the framework at A and leaves it at B. If the difference of potential between C and D is $\frac{1}{3}$ volt, find the magnitude of this current.

8. A coil of 10 turns and mean radius 8 cm. is held in the magnetic meridian, and a small magnet suspended at a point on the axis of the coil at a distance of 6 cm. from the centre is deflected through an angle of 10° when a current traverses the coil. If the moment of the magnet is 25, and the intensity of the earth's horizontal force is 0·18 dyne, find the strength of the current (tan 10° = 0·1763).

9. Find the intensity of the magnetic field inside a solenoid of length 30 cm. containing 250 turns of wire of radius 6 mm., when a current of $\frac{1}{2}$ ampere flows through it.

10. Two wires of lengths ·75 cm. and 80 cm. respectively are made of exactly the same material, and when stretched by certain weights the number of beats heard per second when each vibrates in its fundamental mode is 5. When the weights are reversed the number of beats heard is 12. Find the ratio of the weights.

Paper XLII

1. Two masses, M and m lb., of which M is the greater, are connected by a weightless string passing over a smooth pulley, and initially M is held at a height of h feet above the ground. The system is allowed to move, and M strikes the ground; find the coefficient of elasticity if the string is just tight again at the end of the second rebound.

2. The average length of a thread of mercury in a capillary tube is 1·832 cm. and its mass is 0·169 gr. If the density of mercury is 13·596 gr. per c.c., and the surface tension of water is 78 c.g.s. units, find to what height water will rise in the tube when it is held vertically in a vessel of water.

3. A piece of brass is counterpoised by brass weights at 25° C., and is found to weigh 245 gr. If the density of brass at 0° C. is 8·89 gr. per c.c., and its coefficient of cubical expansion is 0·000055, find the real weight of the body, given that 1 litre of air at 0° C. and 760 mm. weighs 1·294 gr., and that the coefficient of expansion of air is 0·00366.

4. A quantity of air occupies 750 divisions of a tube under a pressure of 760 mm. of mercury. The pressure is increased to 764 mm. while the temperature remains constant, and the volume of the air is thereby decreased by 3·94 divisions. Find the coefficient of elasticity of the air. If the ratio of the specific heats of air is 1·406, find what change there would have been in the volume if the pressure had been increased without any loss or gain of heat.

5. The telescope of a spectrometer is in a position to view the slit of the collimator directly. A prism of angle

60° is placed on the table of the instrument with its angle turned away from the slit, and it is found that in order to view the slit by refraction at grazing incidence on one of the faces the telescope has to be turned through an angle of 57° 54′. Find the refractive index of the prism, given that sin 27° 54′ = 0·4682.

6. A thick lens is formed of a cylinder of glass 6 inches long and 3 inches in radius, surmounted at each end by a hemisphere of the same material, of refractive index 1·5. Find the geometrical focus, after refraction through the cylinder, of a pencil proceeding from a point on the axis at a distance of 4 inches from the nearer end.

7. A building contains 500 lamps of 8 candle-power and 100 of 32 candle-power, each lamp absorbing 4 watts per candle. If the lamps are fed by a dynamo, and the difference of potential between its terminals is 230 volts, find the current which must be supplied.

8. A wire 1 mm. in diameter carries a current of 8 amperes and is found to reach a steady temperature of 120° C. The specific resistance of the material is 3×10^{-4} ohms. Find the amount of heat radiated per second by 1 sq. cm. of the surface of the wire at the temperature 120° C., taking the mechanical equivalent of heat as $4{\cdot}2 \times 10^{7}$ ergs per gramme-degree Centigrade.

9. The plates of a charged condenser are joined by a wire, and the circuit includes also a resistance of 1,500 ohms and a coil of self-inductance 5×10^{6} electromagnetic units. If the effective area of the condenser is 10 square metres, prove that the discharge is oscillatory provided the distance between the plates is greater than 1 cm.

10. A wire of metal of density 7·8 gr. per c.c. has a length of 1 metre and a radius 0·2 mm. A weight of 10 kg. is hung on the wire, and the elongation produced is 5·02 mm. Find from these data the velocity of sound in a solid block of the metal.

Paper XLIII

1. A spiral spring carrying a weight of 150 gr. is stretched by 1 cm. when an additional weight of 10 gr. is added. Find the time of a vertical oscillation.

2. Two masses, of 14 lb. and 18 lb. respectively, are attached to the ends of a rope which passes over a smooth pulley. Initially they are both at a height of 16 feet above the floor, and the string is tight; they are then let go, and the rope breaks 2 seconds later. Find the time that elapses between the moments when the two masses strike the floor.

3. A mass of 200 kg. slides down a rough inclined plane of 1 in 40, and reaches the bottom with a velocity of 500 cm. per sec., the length of the plane being 200 metres. Prove that the heat developed by friction is sufficient to raise 10 gr. of water from 13° C. to the boiling-point (g = 981 cm. per sec. per sec., and the mechanical equivalent $= 42 \times 10^6$ ergs per calorie).

4. A Centigrade thermometer is immersed in liquid up to the mark 30°, and the temperature recorded is 40° C., the temperature of the air being 15° C. If the apparent coefficient of expansion of mercury in glass is 0·000154, prove that the temperature recorded differs from the real temperature by about 0·39° C., the part of the scale not immersed being supposed to be at the temperature of the air.

5. A candle flame is placed at a distance of 20 inches from a screen. Given two convex lenses of focal lengths 9 inches and 12 inches, find the positions in which they must be placed in order to give an image on the screen (1) half, (2) twice, the size of the object.

6. Two prisms, made of different materials, are placed together with one face in contact, and the refracting angles turned in opposite directions. A ray of light passing through them makes angles A, a, B, b, C, c, with the three faces in order. Prove that, if the deviation is a minimum, $\sin A . \sin B . \sin C = \sin a . \sin b . \sin c$.

7. Two wires A_0A_5, B_0B_5, are divided into five equal parts at the points $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$, the resistance of each part and of A_5B_5 being 1 ohm. The corresponding points of each wire are connected by wires, and a battery is placed in A_0B_0. If the current in each cross-wire is the same, prove that the resistances of the cross-wires A_2B_2, A_3B_3, and A_4B_4 are respectively 13, 7 and 3 ohms.

8. Two points A and B are connected by a wire of which the end at A is connected to one terminal of a battery while that at B is connected to one terminal of a receiver, the other terminals of the battery and receiver being earthed. A fault of given earth resistance is found to exist somewhere in the wire. Prove that the current at the receiving end is a minimum when the fault is situated at the middle of the wire.

9. A metal disc of radius 40 cm. rotates 15 times a second about a horizontal axis perpendicular to its plane and in the magnetic meridian, the intensity of the earth's field being 0·18 dyne. Two brushes, one pressing against the circumference of the disc and the other against the axis, are joined by a wire of resistance $\frac{1}{20}$ ohm. Find the strength of the current in the wire.

10. An open organ pipe gives a note of frequency 256 when blown with air at 15° C. What will be the frequency of the note emitted when the temperature of the air rises to 88° C.? Find also the length of the pipe.

[Specific gravities of air and mercury at 0° C. and 760 mm. are respectively 0·001293 and 13·596; the coefficient of expansion of air is $\frac{1}{273}$, and the ratio of the specific heats of air is 1·407.]

Paper XLIV

1. A bead of density 1·03 gr. per c.c. is gently dropped into a beaker of liquid of density 1·025 gr. per c.c. If the depth of the liquid be 20 cm., find how long it will take the bead to reach the bottom.

2. A solid of specific gravity 11·3 is balanced with brass weights of specific gravity 7·8 and is found to weigh 375 gr. If the specific gravity of air is 0·00129, find what error is introduced by neglecting the displacement of the air.

3. Find the reading of a mercury-in-glass thermometer when the bulb and stem up to the zero-mark are exposed to a temperature of 200° C., while the rest of the stem is at the temperature 15° C., the coefficient of cubical expansion of mercury and glass being respectively 0·00018 and 0·000026.

4. Air is compressed into a receiver which stands in a calorimeter, the water equivalent of the latter and its contents being 1,489 gr. The air is then allowed to escape, and under an atmospheric pressure of 75·9 cm. it is found to occupy a volume of 6·52 litres. The calorimeter registers a decrease of $\frac{1}{10}^\circ$ in temperature. Find the mechanical equivalent of heat, given that the density of mercury is 13·596 gr. per c.c., and that $g = 981$ cm. per sec. per sec.

5. Prove that in order that the distance between an object and its image formed by refraction at a convex spherical surface may be a minimum the object must be placed at a distance $\frac{r}{\sqrt{\mu - 1}}$ in front of the surface, r being the radius of the surface and μ the refractive index.

6. In experimenting with a concave spherical mirror, it is found that a shift of 6 cm. in the position of the object causes the image to move through 12·71 cm., while a shift of 12 cm. of the object moves the image through 37·48 cm. Find the focal length of the mirror.

7. A bar magnet of length 7 cm. and moment 140 is placed along one of the arms of a magnetometer, which are set at right angles to the magnetic meridian, with its centre at a distance of 20 cm. from the needle. Calculate the error per cent. in the value of the earth's horizontal force obtained on the assumption that the force on the needle varies inversely as the cube of its distance from the centre of the magnet.

8. A current alternating 30 times a second is sent through a circuit consisting of two branches. In one branch there is a lamp of resistance 20 ohms and a coil of self-induction L, and of negligible resistance. The other branch contains 26 lamps similar to the first, and it is found that all the lamps glow with equal brilliancy. Find the value of L.

9. The poles A, B of a battery of E.M.F. 4 volts and resistance 2 ohms are joined by two wires ACB, $ADEB$. CD and CE are connected. The resistances of AC and CE are each 2 ohms, of AD and DB, 3 ohms, and of CB, 4 ohms. If the point E is such that no current flows along CD, find the currents in AC and AD.

10. A string passing over a smooth pulley carries a weight at one end, while its other end is attached to one of the prongs of a tuning-fork. When the weight is 24 gr. the string divides into four segments when the fork is set in vibration. Find what weight must be attached to the string in order that the latter may vibrate in three segments.

Paper XLV

1. An equilateral triangle of side 12 feet is held under water with one side horizontal and the opposite vertex downwards. If the average pressure over the triangle be 25 lb. per square inch, find the depth of the vertex below the surface, given that the height of the water barometer is 34 feet and that a cubic foot of water weighs 1,000 oz.

2. The weight of a flask when empty is 27·492 gr. When filled with water of density 0·9886 gr. per c.c. it weighs 77·275 gr.; when filled with a liquid whose density is to be determined it weighs 69·483 gr. Find the density of the liquid, given that 1 c.c. of air weighs 0·001293 gr.

3. Two glass globes are filled, one with dry hydrogen and the other with hydrogen saturated with water vapour, both at a temperature of 20° C. and pressure 775 mm. Find the difference between the weights of the globes, given that the pressure of aqueous vapour at 20° C. is 17·5 mm., that the volume of each globe is 500 c.c., the density of air, 0·001293, of hydrogen, 0·000088, and of water vapour, 0·000796 gr. per c.c.

4. A cylindrical vessel made of metal 8 mm. thick is filled with melting ice. If its external length and diameter are respectively 40 cm. and 20 cm., and its thermal conductivity is 0·18 unit, find how much ice will be melted per hour if the vessel is kept surrounded by water at 99·6° C., the latent heat of fusion of ice being 80.

5. Prove that the squares of the radii of the rings observed by reflection from a glass plate in contact with a lens form an arithmetical progression. If the radius of the lens be 100 cm., and the radii of the first and twentieth rings

observed at normal incidence with red light be respectively 0·2628 cm. and 1·1233 cm., find the wave length of red light.

6. Three convex lenses of equal power are placed on the same axis and separated by intervals equal to half the focal length of each; prove that the focal length of the combination is $\frac{4}{5}$ of the focal length of each lens.

7. The resistances of the arms of a Wheatstone's bridge in order are 3, 2, 1 and 2 ohms respectively. Prove that, if the E.M.F. of the battery is 1 volt, and the resistances of the battery and galvanometer are 1 ohm and 5 ohms respectively, the currents through the galvanometer in the two cases in which the diagonals containing the galvanometer and battery are interchanged are connected by the relation

$$\frac{1}{C} - \frac{1}{C_1} = 1.$$

8. A condenser is charged by a battery and then discharged through a ballistic galvanometer, the first swing being 54 scale divisions. The resistance of the galvanometer is 4,850 ohms, and the time of oscillation of its coil is 3·1 seconds. The galvanometer is then shunted, and a current from the battery passed through it, the resistances including the battery being 3,600 ohms, and the shunt resistance being such that only $\frac{1}{576}$ of the battery current passes through the galvanometer. If the steady deflection observed be 84 divisions, find the electromagnetic measure of the capacity of the condenser.

9. A disc of copper of radius 15 cm. rotates about an axis perpendicular to its plane 20 times a second, the plane of rotation being at right angles to the magnetic meridian; find the E.M.F. between the centre of the disc and the circumference, given that the earth's horizontal intensity is 0·18 dyne.

10. Find the frequency of the note emitted by a closed organ pipe 1 metre long, blown with air at 12° C. The coefficient of expansion of air is 0·003665, the ratio of its specific heats is 1·407, its density at 0° C. and 760 mm. is 0·001293 gr. per c.c., while the specific gravity of mercury is 13·596, and the acceleration due to gravity is 981 cm. per sec. per sec.

Paper XLVI

1. A sphere of mass 200 kg. and radius 5 cm. is attached to a wire of length 70 cm., and executes oscillations about a horizontal axis. Neglecting the mass of the wire, find the time of oscillation (g = 981 cm. per sec. per sec.).

2. A piece of salt weighing 26·87 gr. in air weighs 11·68 gr. in a saturated solution of salt. A piece of metal weighing 187·2 gr. in air and 163·2 gr. in water is found to weigh 158·16 gr. in the solution. Find the density of the salt and of the solution.

3. A mercury thermometer has a stem 20 cm. long, the internal radius of which is 0·035 cm. If the thermometer can only be between the temperatures $-10°$ C. and $110°$ C., find the capacity of the bulb, given that the coefficient of expansion of mercury relative to glass is 0·000154.

4. Given that the thermal conductivity of iron is 0·15 c.g.s. unit, find how long it will take for sufficient heat to pass by conduction through a metre cube of the metal, one face of which is maintained at 30° C., to melt 10 kg. of ice at 0° C. at the opposite face, assuming that all the heat that reaches this face is taken up by the ice (latent heat of fusion of ice = 80).

5. It is required to construct a compound lens of focal length 10 cm., using two lenses of crown and flint glass respectively, so that the lens may be achromatic for two given colours in the spectrum. If the refractive indices for these colours are respectively 1·524 and 1·547 for crown glass, and 1·627 and 1·671 for flint glass, find the focal lengths of the two lenses.

A luminous point is placed on the axis of a prolate spheroidal surface which is silvered at one end of the axis. point be midway between the reflecting portion and ıtre of the surface, and the image be at the other end of the axis, prove that the eccentricity of the surface is $\frac{1}{\sqrt{5}}$.

7. An electrometer whose needle is charged to 100 volts, and one pair of whose quadrants is earthed, shows a deflection of 95 mm. on the scale when the other pair of quadrants is charged by a Clark cell of E.M.F. 1·434 volts. Find the E.M.F. of a cell which gives a deflection of 70 mm., assuming that the potential is proportional to the deflection; and show that the true value differs from this by about 0·2 per cent.

8. Given that the heat of combination of 1 gramme-molecule of zinc with sulphuric acid is 38,000 calories, and that of 1 gramme-molecule of copper with sulphuric acid is 12,500 calories, and that 193,000 coulombs are required to decompose 18 grammes of water, find the E.M.F. of a Daniell cell, taking the mechanical equivalent of heat as 42×10^6 ergs per calorie.

9. Five points, A, B, C, D, E, are joined in pairs by wires. The resistances of AB, BC, CD, DE, EA, are each 1 ohm, and those of AC, AD, BD, BE and CE, are each 2 ohms. A current enters the framework at A and leaves it at C. Find the equivalent resistance of the network between the two points.

10. A solid bar of metal 1 metre in length and 0·8 cm. in diameter is vibrating longitudinally. If the density of the metal is 7·8 gr. per c.c., and the value of Young's modulus for it is $1{\cdot}28 \times 10^{12}$ c.g.s. units, find the pitch of the note emitted.

Paper XLVII

1. A solid cone of angle 60° floats in a liquid with its vertex above the surface and its base touching the surface. If the density of the liquid is 8, prove that the density of the cone is about 5·17.

2. A rod of radius 0·5 cm. and length 20 cm., weighing 40 gr., has a ball at each end of radius 2·5 cm. and weighing 100 gr. If it be suspended by a wire through its middle point so as to execute small oscillations in a horizontal plane, find the periodic time, given that the couple required to twist the suspending wire through one degree is $1{\cdot}2 \times 10^4$ c.g.s. units.

3. Determine the form of the adiabatic curves of a substance for which the product of the pressure and specific volume is a linear function of the temperature.

4. The latent heat of steam at 100° C. is 536. Find the amount of heat spent in internal work when water at 100° C. is converted into steam at the same temperature, given that a kilogramme of water when converted into steam at atmospheric pressure occupies a volume of 1·65 cubic metres.

5. A simple astronomical telescope has a magnifying power of 6, and when focussed on a distant object the distance between the lenses is 105 cm. When focussed on a nearer object the distance between them is found to be 106 cm., Find the distance of the object.

6. If a luminous point be placed at the vertex of a cone of angle 60°, compare the intensities at points of a circular section of the cone of radius 1 foot, and an elliptical section of semi-axes 2 and 3 feet, the points in question lying on the axis of the cone.

7. A coil of 250 turns and mean radius 20 cm. has its terminals connected to a galvanometer through a certain resistance. It is held at right angles to the magnetic meridian, and when rotated through two right angles it is found that the throw of the galvanometer corresponds to the passage of one-millionth of a coulomb of electricity through the coil. Find the resistance in the circuit. ($H = 0{\cdot}18$ dyne.)

8. A wire is formed into a circle of radius 40 cm. Two points on the circumference distant 50 cm. apart are connected to a battery of E.M.F. 2 volts and resistance 10 ohms. If the resistance of the wire is 15 ohms per metre, find the current in the battery.

9. Find the force of attraction between two circular plates of radius 10 cm., at a distance of 3 mm. from each other, the difference of potential between them being 1,800 volts.

10. Assuming that the strings of a violin are of exactly the same thickness, length and material, and that they emit the notes G_1, D, A and E^1 when bowed, compare the tensions in the strings, given that the frequency of the note C is 256.

Paper XLVIII

1. A mass of 250 kg. hangs by a spiral spring, and when another 10 gr. is added it is found that the length of the spring is increased by ·75 cm. Find the time of a vertical oscillation of the spiral.

2. A thread of mercury of length 13·1 cm. in a capillary tube is found to weigh 0·44 gr., and when the tube is placed in water the liquid rises in it to a height of 5·52 cm. If the density of mercury at the temperature of the experiment is 13·56 gr. per c.c., find the surface tension of water.

3. The volume of 1 gr. of hydrogen at 0° C. and 760 mm. is 11·16 litres; the density of mercury at 0° C. is 13·596 gr. per c.c., and the acceleration due to gravity is 981 cm. per sec. per sec.; taking the mechanical equivalent of heat as 42×10^6 ergs, calculate in heat units the difference of the specific heats of hydrogen.

4. Water is contained in a glass vessel with a fine graduated stem. The coefficient of cubical expansion of glass is 0·000026, and the volume of 1 gramme of water at a temperature $(t + 4)$ °C. is given by $1 + at^2$, where $a = 7 \times 10^{-6}$. Find the temperature at which the water in the vessel will have its least apparent volume.

5. Two lenses, each of focal length 15 inches, are placed coaxially at a distance 20 inches apart, and a sphere of glass of the same refractive index 1·5 is placed midway between them. An object is placed at a point 5 inches behind the first lens, and an image is formed at a point 5 inches in front of the second. Find the radius of the sphere.

6. Prove the formula $\sin \theta = Nn\lambda$ for normal incidence on a diffraction grating, where 2θ is the angle between the

spectra on each side of the line of direct vision, N is the number of lines per unit length of the grating, and λ is the wave-length of the light used. Hence find λ, given that the deviation of the first spectrum observed with a grating containing 1108·4 lines per cm. is 3° 44′ 53″ (sin 3° 44′ = 0·0651129; sin 3° 45′ = 0·0654031).

7. A circuit contains a condenser and a key by means of which the condenser can alternately be charged by a battery or discharged through a ballistic galvanometer. Prove that, whatever resistance there is in the galvanometer circuit, the throw of the galvanometer is always the same.

8. A current from 4 Daniell cells in series is passed through solutions of silver nitrate and sulphuric acid, and after 7 minutes it is found that ·1 gr. of zinc has been dissolved in each cell. Find how much sulphuric acid has been decomposed, how much silver has been deposited, and the strength of the current, given that one ampere deposits 0·001118 gr. of silver per second, and that the atomic weights of hydrogen, oxygen, copper, zinc, silver and sulphur are respectively 1, 16, 63·5, 65, 108 and 32.

9. A unit point charge is placed 3 cm. in front of an infinite conducting plane. Prove that the total charge induced on the portion of the plane bounded by the circumference of a circle of radius 4 cm., whose centre is the foot of the perpendicular from the charged point, is numerically equal to $\frac{2}{5}$.

10. Assuming that the frequency of the note given out by a rod of length l, tension T, and linear density e, is $\frac{1}{2l}\sqrt{\frac{T}{e}}$, find the tension in a rod of length 40 cm. and radius 1 mm., which when plucked gives a note of frequency 256, the density of the rod being 8·8 gr. per c.c.

Paper XLIX

1. A wheel of radius 20 cm. and mass 2 kg. is free to rotate about a horizontal axis through its centre. A string passing over it carries a mass of 50 gr. at one end. Find the velocity of this mass after it has fallen for 3 seconds.

2. Find the force required to prevent the contraction of a cylindrical bar of copper 5 mm. in radius in cooling from 50° C. to 10° C., given that the coefficient of linear expansion of copper is 0·000018, and that Young's modulus for the metal is 12×10^{11} dynes per sq. cm.

3. Atmospheric air consists of oxygen and nitrogen in the proportion of 3 to 10 by mass. Assuming that hydrogen, oxygen and nitrogen are perfect gases for which the equation $pv = Rt$ holds, find the value of R for air; given that the density of hydrogen at 0° C. and 760 mm. is 0·00009, the density of mercury at 0° C. is 13·596, the atomic weights of oxygen and nitrogen 16 and 14 respectively, and $g = 981$ cm. per sec. per sec.

4. If P, V, T are the values of the ratios of the pressure p, specific volume v, and temperature t, of a substance, to the critical values of these quantities respectively, show that the equation $\left(p + \frac{a}{v^2}\right)(v - b) = Rt$, of Van der Waals, becomes $\left(P + \frac{3}{V^2}\right)\left(V - \frac{1}{3}\right) = \frac{8}{3}T$.

5. A sphere of glass 6 inches in radius has a concentric spherical cavity of radius 2 inches. An object is placed at a distance of 10 inches from the centre. If the refractive index is 1·5, find the position of the image.

6. Newton's rings are formed between a lens and a plane glass surface. If the diameter of the fifth ring is 0·8 cm. when light of wave-length 0·0000589 cm. is used, and the direction of the light passing through the air film makes an angle of 30° with the normal, find the radius of the lens.

7. A soft iron wire of length 30·2 cm. and radius ·5 mm. is placed inside a solenoid with one end at a distance of 8·5 cm. east of the centre of a magnetometer. A current is passed through the solenoid, and the deflection of the needle is 45°. Assuming that the end of the wire is in the same horizontal plane with the needle, and that the other end is far enough away for its effect to be neglected, find the intensity of magnetisation ($H = 0{\cdot}18$ dyne).

8. A circular proof-plane of radius r is laid on the surface of a charged spherical conductor of radius R, which is large compared with r. If the capacity of the proof-plane is comparable with r and the plane carries away its whole charge, prove that the loss of electrical energy in removing it from the conductor when the latter has unit charge is approximately $\frac{1}{4R}\left(\frac{r}{R}\right)^2$.

9. A condenser consisting of two parallel plates of area 2,000 sq. cm., separated by a distance 5 mm. in air, is charged by a battery and discharged through a galvanometer fifty times a second by means of a vibrating fork, the deflection on the scale being 55 mm. The battery is then connected to a resistance of 539 ohms and also a 1-ohm resistance in series with it, to the ends of which the galvanometer is connected through a resistance of 10^5 ohms. The resistance of the galvanometer is 860 ohms, and the deflection observed is the same as before. Find from this experiment the ratio of the electrostatic measure of capacity to the electromagnetic measure.

10. Prove that the pitch of the whistle of a train travelling at the rate of $46\frac{1}{8}$ miles an hour falls from C to B on the diatonic scale as it passes an observer, assuming the velocity of sound in air to be 1,100 feet per second.

Paper L

1. A compound pendulum has two knife-edges distant 7·95 and 31·55 inches respectively from its centre of gravity. The pendulum is suspended from each knife-edge in turn, and the observed times for 300 oscillations are respectively 493·2 and 577·6 seconds. Find the value of g from this experiment.

2. A wire 3 metres long and 0·8 mm. in diameter is stretched by hanging on a weight of 10 kg. If the elongation produced is 0·45 mm., find the value of Young's modulus for the material.

3. If the pressure, specific volume and absolute temperature of a substance be connected by the relation $pv = Rt^3$, prove that the difference of the specific heats at constant pressure and constant volume is equal to $9Rt^2$.

4. An engine consuming a ton of coal every two hours can draw a train at the rate of 40 miles an hour against resistances equal to the weight of 2 tons. If the combustion of 1 lb. of coal generates sufficient heat to raise 80 lb. of water from the freezing- to the boiling-point, find the efficiency of the engine given that the mechanical equivalent of heat is 1,390 foot-pounds per pound-degree Centigrade.

5. A luminous circular disc of area 10 sq. mm. is placed in front of a magnifying glass at a distance of 3 cm. from it, and an image is formed on a screen at a distance of 12 cm. on the other side of the lens. Prove that the diameter of the image will be nearly 14·3 mm.

6. A prism of angle 42° 30′ is placed on the table of a spectroscope, and the spectrum formed by cadmium vapour is viewed through it. In the two positions for minimum

deviation of the red line the telescope reads 209° 18′ and 150° 10′ respectively, while the readings for the blue line are 209° 14′ and 149° 15′. Calculate the refractive indices of the prism for red and blue light, and find the dispersive power of the prism, given that the refractive index for sodium light is 1·627.

sin 21° 15′ = 0·362438; sin 36° 2′ = 0·588256;
sin 36° 14′ = 0·591075; sin 36° 15′ = 0·591310.

7. Two horizontal rods of brass are placed parallel to each other at a distance of 50 cm. apart, and another rod slides along them in a direction parallel to itself with a velocity of 75 cm. per sec. If the intensity of the earth's vertical force is 0·44 dyne, calculate the E.M.F. between the ends of the fixed rods.

8. A magnetic shell of strength 20 has the form of the zone of a sphere of radius 10 cm. situated between the latitudes 30° and 60°. Prove that the magnetic force at the centre of the sphere is π dynes.

9. A coil of radius 16 cm. having 210 turns has its terminals connected to a ballistic galvanometer through a resistance of 1,000 ohms, and is initially held at right angles to the magnetic meridian. It is then suddenly rotated through two right angles about the vertical diameter, and the throw of the galvanometer is 27 scale divisions. To standardise the galvanometer the terminals of a 4-volt battery are connected to a resistance of 1,000 ohms and also a resistance of 1 ohm in series with it. The ends of the latter are connected through a resistance of 4,250 ohms to the galvanometer, which then shows a steady deflection of the same amount. If the time of swing of the galvanometer coil be 4·7 seconds, find the value of H.

10. Given that the density of air at 0° C. and 760 mm. is 0·001293 gr. per c.c., the density of mercury under the same conditions 13·596 gr. per c.c., the coefficient of expansion of air 0·003665 and the ratio of the specific heats of air 1·407 calculate the velocity of sound in air at 15° C. and 745 mm (g = 981 cm. per sec. per sec.).

48, 36; 35·3, 47 ergs.
597; 2984000 ergs (approx.).
220, 484; 44 × 10^5 ergs; 968 × 10^4 ergs.

VII.—CURRENT ELECTRICIT

·123 gr.
24 min. 51 sec.
·253 gr.; 1·28 amperes.
·097 gr.; ·41 ampere.
3 hrs. 43 min. 37 sec.
·283 gr.; ·576 ampere.
·341 gr.
·197 gr.; ·671 gr.
·751 gr.
·508.
·847.
·04.
·143.
·0086 ampere.
·306 ampere.
·135 gr.
17·54.
·32 ampere.
1·5 volts.
55 ohms.
$\frac{1}{8}$ ampere.
5 volts.
110 volts.
1·6 volts.
2 volts; ·75 volt; 1·25 volts.
·22 ampere.
18.
1·375 volts; 1·25 volts.
12·5 ohms.
93·8 ohms.
17 ohms.
4·6 ohms.
45 ohms.
14 ohms; $3\frac{3}{7}$ ohms.
8 ohms; 4 ohms.
·17 ampere; ·163 ampere; ·245 ampere.
1·435 amperes.
1·9 volts; 2·6 ohms.
1·15 amperes; ·18 ampere; ·81 ampere.
1·63 amperes; 1·08 amperes; ·55 ampere.
11.
2010 ohms.

47. 150 ergs.
48. 27343$\frac{3}{4}$ ergs.
49. 600 units; 1·33.
50. 8·97; 2141 units.

43. 6 ohms.
44. $9\frac{1}{5}$ ohms.
45. 4 parallel rows; 2·1 amperes.
46. 1·94 amperes.
47. ·55 ampere; ·3 ampere.
48. 1·1 amperes.
49. $1\frac{1}{4}$ ohms.
50. ·29 ampere; ·15 ampere; ·10 ampere; ·54 ampere.
51. 1·07 volts.
52. ·12 ampere.
53. 17.
54. 1·5 volts.
55. 1·36 gr.; 5·01 volts.
56. 17·7 ohms.
57. ·00005 ohm.
58. 15$\frac{3}{4}$ ohms.
59. 130 cm.
60. 1 ohm.
61. $\frac{3}{5}$ ohm.
62. $2\frac{2}{5}$ ohms; $\frac{4}{5}$ volt.
63. $\frac{1}{11}$ ampere; $\frac{1}{110}$ ampere.
65. 2·4 volts; ·8 volt; ·6 volt.
66. 15 × 10^8 ergs.
67. 6 × 10^8 ergs.
68. 24° C.
69. 1070·4; 14 min. 52 sec.
70. ·68 gr.
71. 60°.
72. 20.
73. ·02 ampere.
74. ·15 ampere.
75. 1·4 volts; $\frac{1}{2}$ ohm.
76. $\frac{1}{4}$ ampere
77. $\frac{1}{4}$ ampere.
78. ·9 volt.
79. ·4 ohm.
81. 11 ohms.
82. 1·5 volts; 1 ohm.
83. $27\frac{7}{8}$ ohms.
84. 37·3 cm.
85. 180 ohms.
86. 255 ohms.
87. 1 volt.
88. 1·15 volts.

89. $2\frac{2}{11}$ ohms.
90. 336 ohms.
91. $26\frac{1}{2}$ ohms.
92. 40·2 calories.
93. 1·5 ohms.
94. $\frac{1}{2}$ ohm.
95. ·28 ampere; ·224 ampere; ampere.
96. 1·4 amperes.
97. 3 parallel rows.
99. 2 ohms.

VIII.—SOUND

2. 32905 cm. per sec.
3. 343·94 metres per sec.
4. 35·6° C.
5. 1261·2 metres per sec.
6. 1·61.
7. 295×10^8.
8. 33216 cm. per sec.
9. 11·1 in.
10. 1414·2 metres per sec.
11. 1·405.
12. $14{\cdot}25 \times 10^5$.
13. 20×10^{10} dynes per sq. cm.
14. 4835 metres per sec.
15. 5043 metres per sec.
16. 5494 metres per sec.
17. 17078 ft. per sec.
18. 45×10^8 poundals per sq. ft.
19. 4·6 mm.
20. 1035·28 metres per sec.
21. $12\frac{1}{2}$ kg. wt.
22. 300·6 kg. wt.
23. 55·4.
24. 165·7.
25. 80 kg.; 45 kg.
26. 282·8.
27. C.; frequency 128.
28. 150 cm.
29. 405 kg.; 720 kg.
30. 3 : 2.
31. $\sqrt{6}$: 2.
32. 57·46 kg. wt.
33. 45 : 18 : 10.
34. 150.
35. 21 lb.
36. 80; 40.
37. 100; 200; 300.
38. $33\frac{1}{3}$, $66\frac{2}{3}$; $16\frac{2}{3}$, 50.
39. 33440 cm. per sec.
40. 322.
41. 570.
42. 30° C.
43. 797·9.
44. $46\frac{7}{8}$ miles per hr.
45. 240 (nearly).
46. 172·4.

THE ABERDEEN UNIVERSITY PRESS LIMITED